4.6 Motion Along a Line (1-Dimension)

Position, Velocity, and Acceleration

Position

$s(t)$ the position of an object moving along a line at time t
Same as $x(t)$ for some physics classes

Displacement from $t = a$ to $t = b$

$$displacement = s(b) - s(a)$$

Displacement the change in position of the object

Average Velocity (slope of secant line)

$$v_{avg} = \frac{displacement}{time\ elapsed} = \frac{\Delta s}{\Delta t} = \frac{s(b)-s(a)}{b-a}$$

Instantaneous Velocity (slope of tangent line)

$$v(t) = \frac{ds}{dt} = \lim_{h \to 0} \frac{s(t+h)-s(t)}{h} = s'(t)$$

Average Acceleration (slope of secant line)

$$a_{avg} = \frac{change\ of\ velocity}{time\ elapsed} = \frac{\Delta v}{\Delta t} = \frac{v(b)-v(a)}{b-a}$$

Instantaneous Acceleration (slope of tangent line)

$$a(t) = \frac{dv}{dt} = \lim_{h \to 0} \frac{v(t+h)-v(t)}{h} = v'(t)$$

$$a(t) = v'(t) = s''(t)$$

Instantaneous Speed

$$speed = \left|\frac{ds}{dt}\right| = |v(t)|$$

Speed is scalar and has no direction, only the magnitude

Ex. A ball is thrown directly upwards with an initial velocity of 20 m/s from the top of a building 10 m above the ground. The position, in metres, of the ball above the ground after t seconds is given by the function $s(t) = -5t^2 + 20t + 10$.

a. Find the ball's displacement from $t = 0$ to $t = 2$.

$s(0) = 10 \qquad s(2) = -20 + 40 + 10 = 30$

$d = s(2) - s(0) = 30 - 10 = 20$ m

b. Find the ball's average velocity from $t = 0$ to $t = 2$.

$v_{avg} = \frac{s(2)-s(0)}{2-0} = \frac{20}{2} = 10$ m/s

c. Find the ball's velocity and acceleration functions.

$v(t) = s'(t) = -10t + 20$

$a(t) = v'(t) = -10$

d. Find the ball's position, velocity, speed, and acceleration at $t = 3$.

$s(3) = -5(3)^2 + 20(3) + 10 = 25$

$v(3) = -10(3) + 20 = -10$ m/s

$speed = |v(3)| = |-10| = 10$ m/s

$a(3) = -10$ m/s^2

e. How many seconds will it take the ball to reach the highest point? What is the maximum height?

Maximum height occurs when $v(t) = 0$

$-10t + 20 = 0$

$-10(t - 2) = 0$

$t = 2$ s

$s(2) = -5(2)^2 + 20(2) + 10 = 30$ m

The ball will reach a maximum height of 30 m at 2 seconds.

Position, Velocity and Acceleration Graphs

The following explains how the sign of the object's position, velocity, and acceleration determines how the object moves

Position

If $s > 0$, the object is on the positive side of the s-axis
If $s < 0$, the object is on the negative side of the s-axis

Velocity

If $v > 0$, the object is moving in the positive direction
If $v < 0$, the object is moving in the negative direction
If $v = 0$, the object is at rest

If v changes sign, the object changes direction

Acceleration

If $a > 0$, v is increasing
If $a < 0$, v is decreasing

This is different from the **speed** of the object

Velocity and Acceleration (the signs of v and a are the same)

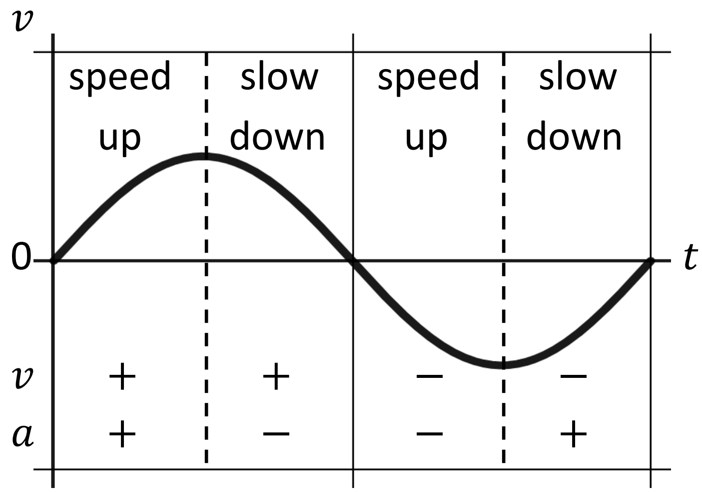
If $a > 0$ and $v > 0$ or $a < 0$ and $v < 0$,
the **speed** of the object is **increasing**.

Velocity and Acceleration (the signs of v and a are opposite)

If $a > 0$ and $v < 0$ or $a < 0$ and $v > 0$,
the **speed** of the object is **decreasing**.

Velocity-Time Curve

A summary of the information above



Ex. An object is moving along a horizontal line. Its position as a function of time is given by $s(t) = t^3 - 3t^2 + 1$, $t \geq 0$ where s is in metres and t is in seconds.

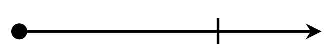
a. Find the velocity and acceleration functions of the object.

$$v(t) = s'(t) = 3t^2 - 6t$$

$$a(t) = v'(t) = 6t - 6$$

b. At what time does the object change direction?

$$v(t) = 0$$
$$3t^2 - 6t = 0$$
$$3t(t - 2) = 0$$
$$t = 0, 2$$



t	0		2	
v	0	$-$	0	$+$

$v(t)$ changes sign at $t = 2$, ∴ object changes direction at $t = 2$

c. During which time intervals is the velocity of the object increasing?

$a(t) = 0$
$6t - 6 = 0$
$t = 1$

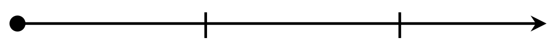
t	0		1	
a	$-$	$-$	0	$+$

$6t - 6 > 0$
$6t > 6$
$t > 1$

$\therefore t > 1$

d. During which time intervals is the speed of the object decreasing?

speed of object is decreasing when the signs of v and a are opposite



t	0		1		2	
v	0	$-$	$-$	$-$	0	$+$
a	$-$	$-$	0	$+$	$+$	$+$

$\therefore 1 < t < 2$

e. Find the total distance travelled by the object during the first 4 seconds.

From b, the object changes direction at $t = 2$.

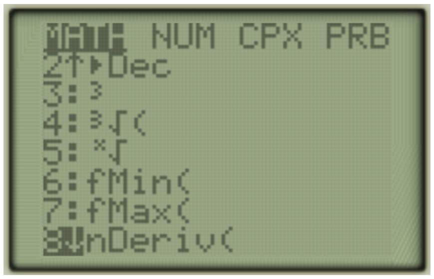
$d_{0to2} = |s(2) - s(0)| = |-3 - 1| = 4$
$d_{2to4} = |s(4) - s(2)| = |17 - (-3)| = 20$
total distance $= 4 + 20 = 24$

$\therefore$ total distance travelled in the first 4 seconds is 24 m

Graphing Calculator Example to Find Slope

Ex. Find the slope of the tangent line to the curve $y = x^2 - 2$ at $x = 2$.

Method 1: nDeriv()

1.


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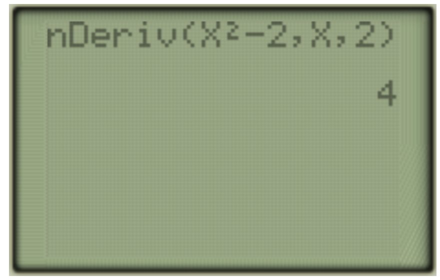
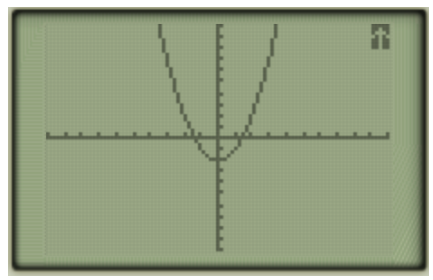


Method 2: Graph and Calculate

1.


2.


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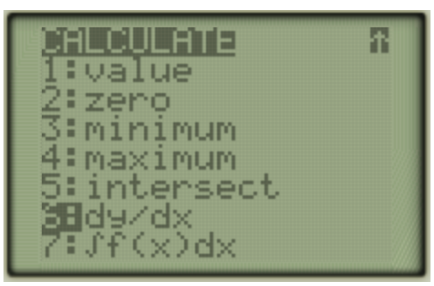

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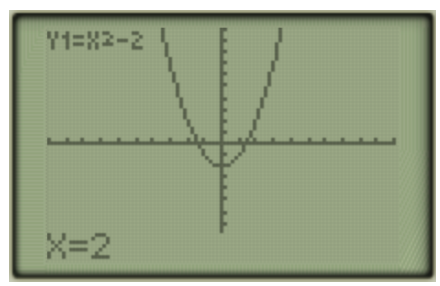

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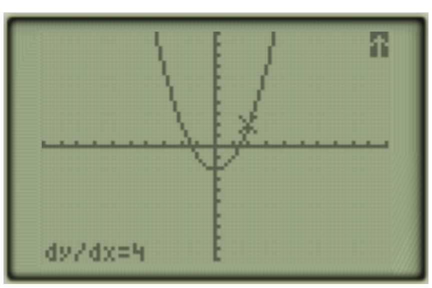

4.6 Homework:

Motion Along a Line (1-Dimension) Worksheet