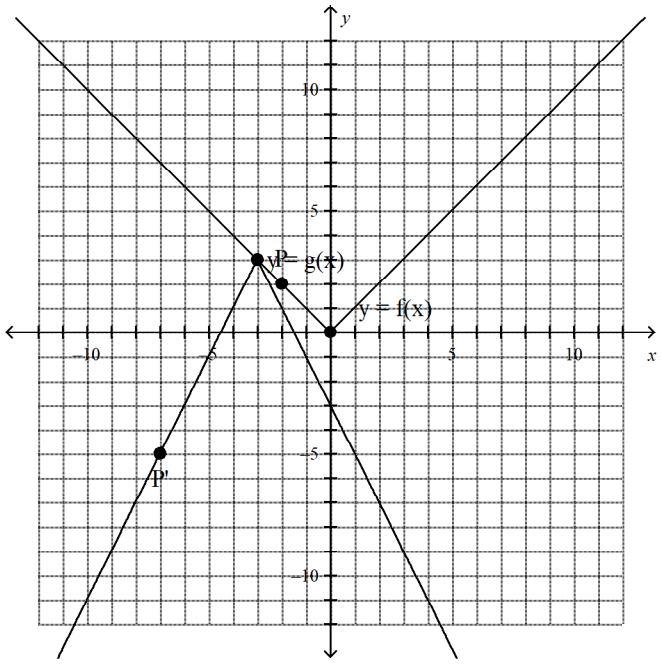
Exam Version: A

Pre-Calculus 12 Final Exam (2025)

Select best answer. If answer is not shown, fill in option "E" on your answer sheet.

1. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a combination of transformations. Corresponding points are labelled. What is an equation of the image graph in terms of the function f?



A. $y+3 = -4f(\frac{1}{2}(x-3))$

B. $y+3 = -\frac{1}{2}f(4(x-3))$

C. $y-3 = -\frac{1}{2}f(4(x+3))$

D. $y-3 = -4f(\frac{1}{2}(x+3))$

2. Determine all restrictions for the expression: $\dfrac{\sec x}{4\sin^2 x - 1}$

A. $\sin x \neq \pm\frac{1}{4}$

B. $\sin x \neq \pm\frac{1}{2}$

C. $\cos x \neq 0$, $\sin x \neq \pm\frac{1}{4}$

D. $\cos x \neq 0$, $\sin x \neq \pm\frac{1}{2}$

3. The function $y = f(x)$ has domain $-3 \le x \le 2$ and range $-2 \le y \le 2$. What are the domain and range of $y - 3 = f(x+5)$?

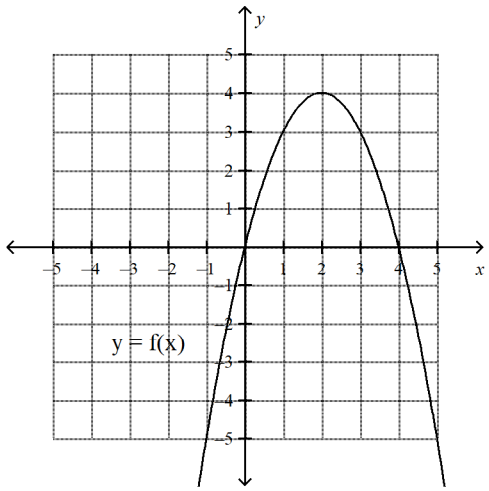
A. domain: $-8 \le x \le -3$
range: $1 \le y \le 5$

B. domain: $0 \le x \le 5$
range: $2 \le y \le 5$

C. domain: $-8 \le x \le -3$
range: $2 \le y \le 5$

D. domain: $0 \le x \le 5$
range: $1 \le y \le 5$

4. For the graph of $y = f(x)$ shown below, what are the domain and range of $y = \sqrt{f(x)}$?



A. domain: $x \le 0$ or $x \ge 4$;
range: $0 \le y \le 2$

B. domain: $x \le 0$ or $x \ge 4$;
range: $y \ge 0$

C. domain: $0 \le x \le 4$;
range: $y \ge 0$

D. domain: $0 \le x \le 4$;
range: $0 \le y \le 2$

5. The graph of $y - 2 = \sqrt{x + 2}$ is translated 4 units left and 5 units up. What is an equation of the image graph?

A. $y - 3 = \sqrt{x + 2}$

B. $y - 5 = \sqrt{x + 4}$

C. $y + 3 = \sqrt{x - 2}$

D. $y - 7 = \sqrt{x + 6}$

6. The graph of $y = f(x)$ is stretched horizontally by a factor of 5, and reflected in the x-axis. What is the equation of the image graph in terms of f?

A. $y = -f(5x)$

B. $y = -\frac{1}{5}f(x)$

C. $y = -f(\frac{1}{5}x)$

D. $y = -5f(x)$

7. Solve: $27^{x-5} = 9^{x+3}$

A. $x = \frac{1}{21}$

B. $x = 21$

C. $x = -\frac{1}{21}$

D. $x = -21$

8. The point M$(-2, 9)$ lies on the graph of $y = f(x)$. What are the coordinates of its image M on the graph of $y = f^{-1}(x - 6)$?

A. $(15, -2)$

B. $(9, -2)$

C. $(-8, 15)$

D. $(3, -2)$

9. Divide: $(3x^2 - 31x + 51) \div (x - 8)$

A. $3x + 51$ R5
B. $-3x + 8$ R5
C. $3x - 7$ R(–5)
D. $-3x - 32$ R(–5)

10. A circle has a radius of 4 cm. If the length of arc AB is 3π cm, determine the measure of the central angle in radians.

A. $\frac{3\pi}{4}$
B. $\frac{4\pi}{3}$
C. $\frac{3\pi}{2}$
D. 3π

11. Given the functions $f(x) = x + 3$ and $g(x) = \frac{1}{x-3}$, what is the simplified form of $(f \circ g)(x)$?

A. $(f \circ g)(x) = \frac{-8}{x}, x \neq 0$
B. $(f \circ g)(x) = \frac{3x-8}{x+3}, x \neq -3$
C. $(f \circ g)(x) = \frac{3x-8}{x-3}, x \neq 3$
D. $(f \circ g)(x) = \frac{1}{x}, x \neq 0$

12. Prom's friend gives him a row of Pascal's triangle and asks which row it comes from. Prom adds the numbers and obtains a sum of 65 536. Which row do the numbers come from?
A. 34
B. 18
C. 17
D. 16

13. Solve: $\log x + \log(x + 21) = 2$

A. $x = 4$
B. $x = -25, 4$
C. $x = 25$
D. $x = 25, -4$

14. Find the sum of the arithmetic series 4 + 7 + 10 + 13 + ... + (3n + 1)
A. $\frac{1}{2}(3n^2 + 5n)$
B. $n + 3n^2$
C. $\frac{1}{2}(3n^2 + n)$
D. $3n - 1$

15. If $(4, -3)$ is a point on the graph of $y = f(x)$, what mus be a point on the graph of $y = f(2x + 10)$?
A. $(-8, -3)$
B. $(-3, -3)$
C. $(3, -3)$
D. $(18, -3)$

16. There are 17 points on the circumference of a circle. How many lines can be drawn to connect all possible pairs of points?

A. 289
B. 136
C. 34
D. 272

17. Given the functions $f(x) = 2x + 4$ and $g(x) = \sqrt{x + 5}$, what is the value of a for which $f(g(a)) = 16$?
A. 31 **B.** 36 **C.** –26 **D.** 35

18. Given $f(x) = 9x^2 + 7x$ and $g(x) = 2 - x$, determine $5g(x) - f(x)$.

A. $45x^2 + 36x - 2$
B. $9x^2 + 6x + 2$
C. $45x^2 + 6x - 5$
D. $-9x^2 - 12x + 10$

19. The sum of the first 25 terms of an arithmetic series is 1600. The sum of the first 26 terms is 1729. The common difference is 5. Determine t_1.

A. $t_1 = 1476$
B. $t_1 = 4$
C. $t_1 = 9$
D. $t_1 = -129$

20. How is the graph of $y = \frac{1}{3}f(x)$ related to the graph of $y = f(x)$?

A. $y = f(x)$ has been compressed vertically by a factor of $\frac{1}{3}$.
B. $y = f(x)$ has been compressed horizontally by a factor of $\frac{1}{3}$.
C. $y = f(x)$ has been expanded vertically by a factor of 3.
D. $y = f(x)$ has been expanded horizontally by a factor of 3.

21. Find the sum of $\sum_{k=a}^{b} 2$

A. $2b$
B. $2(b - a)$
C. $2b$
D. $2b - a$

22. Determine the 1st and 6th terms of a geometric series that has these partial sums $S_4 = -240$, $S_5 = -726$, $S_6 = -2184$

A. $t_1 = -18; t_6 = -486$
B. $t_1 = -6; t_6 = -1458$
C. $t_1 = -18; t_6 = -1458$
D. $t_1 = -6; t_6 = -486$

23. Let A be an angle in standard position such that $0 < A < \frac{\pi}{2}$. If sin A = n and cos A = m, determine an expression for $\sin(\pi + A) + cox(\pi + A)$.

A. - m - n
B. - m + n
C. m - n
D. m + n

24. Express sin x cos x in terms of a single trigonometric function.

A. $\frac{1}{2}\sin 2x$
B. $\frac{1}{2}\cos 2x$
C. 2 sin 2x
D. $2\cos^2 x$

25. Determine the number of terms in an arithmetic sequence with $t_3 = -17.5$ and $t_{10} = -52.5$. The final term in the sequence is -147.5.

A. $n = 18$
B. $n = 20$
C. $n = 13$
D. $n = 29$

26. What is the exact value of the expression $\cos\frac{3\pi}{4}\cos\frac{11\pi}{12}+\sin\frac{3\pi}{4}\sin\frac{11\pi}{12}$?

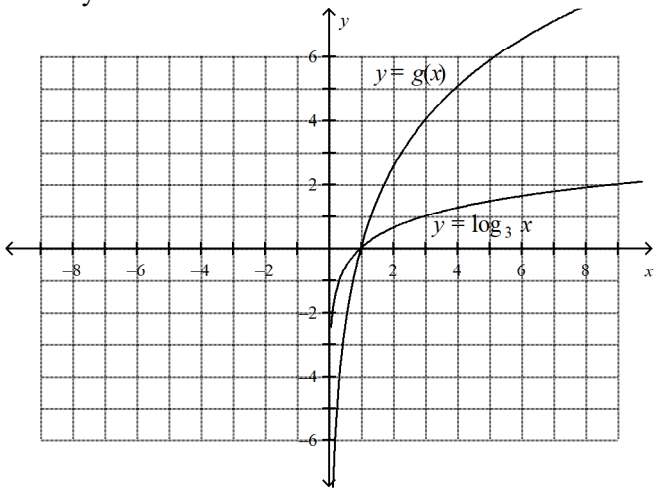
A. $\frac{1}{\sqrt{3}}$

B. $\frac{\sqrt{3}}{2}$

C. $-\frac{1}{\sqrt{3}}$

D. $-\frac{\sqrt{3}}{2}$

27. The graphs of $y=\log_3 x$ and its transformation image $y=g(x)$ are shown. Identify the transformations.



A. A translation of 3 units right and 2 units down

B. A vertical compression by a factor of $\frac{1}{4}$

C. A vertical stretch by a factor of 4

D. A horizontal compression by a factor of $\frac{1}{4}$

28. How many 4-digit numbers greater than or equal to 3000 and less than 8000 can be formed with no repetition in their digits?

A. 5000

B. 5040

C. 120

D. 2520

29. Divide: $(4x^3-16x^2+18x-28)\div(-x+3)$

A. $-4x^2+4x-6$ R10

B. $-4x^2+4x-6$ R(−10)

C. $4x^2-4x+6$ R(−10)

D. $4x^2-4x+6$ R10

30. In a youth hockey league,9 teams compete for the championship. What is the number of ways that the winner, second, third, and fourth place trophies could be awarded?

A. 3024
B. 15 120
C. 504
D. 120

31. Which expression is equivalent to $\frac{\sin\theta + \cos\theta\cot\theta}{\cot\theta}$?

A. $\csc\theta$
B. $\cos\theta$
C. $\sin\theta$
D. $\sec\theta$

32. Solve: $\cos^2 x = \frac{3}{4}$, where $0 \le x < 2\pi$.

A. 0.52, 5.76
B. 1.05, 5.24
C. 0.52, 2.62, 3.67, 5.76
D. 1.05, 2.09. 4.19, 5.24

33. The volume of a shipping box with the shapeof a rectangular prism can be expressed as the polynomial $2x^3 + 11x^2 + 17x + 6$. Each dimension of the box can be expressed as a binomial. Which binomial could represent one dimension of the box?

A. $2x + 1$ **B.** $x + 1$ **C.** $2x + 3$ **D.** $x + 6$

34. A geometric series has$r = \frac{3}{4}$ and $S_\infty = -16$. Determine t_1.

A. $t_1 = 4$
B. $t_1 = -12$
C. $t_1 = -16$
D. $t_1 = -4$

35. Which of the following has30 terms in its binomial expansion?

A. $(-2y)^{29}$
B. $(-4x - 2y)^{29}$
C. $(-4x - 2y)^{30}$
D. $(-4x - 2y)^{31}$

36. A circle has a radius of 12 cm. Determine the area of a sector of the circle that has a central angle of 2.1 radians. (answer in cm^2)

A. 25.2
B. 151.2
C. 215.4
D. 302.4

37. Which equation of a radical function would have the following domain and range?
$\{x | x \ge -6, x \in R\}$; $\{y | y \ge 10, x \in R\}$

A. $y = \sqrt{x - 10} + 6$
B. $y = \sqrt{x + 6} + 10$
C. $y = \sqrt{x + 6} - 10$
D. $y = \sqrt{x + 10} + 6$

38. Simplify: $\frac{2\sin x}{\sin 2x}$

A. 1
B. cos x
C. sec x
D. 2 csc x

39. A coin is tossed 7 times. What is the number of ways the coin can land with 4 heads and 3 tails?

A. 5040
B. 2
C. 144
D. 35

40. What is the coefficient of x^{10} in the expansion of $\left(5x^5 - 3\right)^7$?

A. –127 575
B. –2 373 046 875
C. 76 545
D. 25 515

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PRE-CALCULUS MATHEMATHICS 12
A Summary of Basic Identities and Formulae

Sequence and Series:

$t_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$ $\quad S_n = \frac{n}{2}(a + l)$

$t_n = ar^{n-1}$ $\quad S_n = \frac{a(1-r^n)}{1-r}$ $\quad S_n = \frac{a-rl}{1-r}$ $\quad S_\infty = \frac{a}{1-r}$

Sigma Notation:

$$\sum_{j}^{k} f(j) = f(j) + f(j+1) + \cdots + f(k) \qquad n = k - j + 1$$

Combinations:

$${}_nP_r = \frac{n!}{(n-r)!} \qquad {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \qquad t_{k+1} = {}_nC_k\, a^{n-k}\, b^k$$

Trigonometry:

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$ $\quad \cos C = \frac{a^2+b^2-c^2}{2ab}$

Sum and Difference of Cubes:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $\quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Transformations:

$y = af\big(b(x-c)\big) + d$ $\quad (x, y) \to \left(\frac{1}{b}x + c, ay + d\right)$

Polynomial Division:

$f(x) = d(x)q(x) + r(x)$ $\quad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$

Compound Interest / Exponential Growth and Decay:

$A = P\left(1 + \frac{r}{n}\right)^{tn}$ $\quad A = Pe^{rt}$ $\quad A = A_o x^{\frac{t}{T}}$ $\quad A = A_o e^{kt}$

Pythagorean Identities:

$\sin^2\theta + \cos^2\theta = 1$ $\quad 1 + \tan^2\theta = \sec^2\theta$ $\quad 1 + \cot^2\theta = \csc^2\theta$

Reciprocal and Quotient Identities:

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \csc\theta = \frac{1}{\sin\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Sum and Difference Identities:

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad \sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad \cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$ $\quad \tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$

Even-Odd and Co-function Identities:

$\sin(-\theta) = -\sin\theta$ $\quad \cos(-\theta) = \cos\theta$ $\quad \tan(-\theta) = -\tan\theta$

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$ $\quad \tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$ $\quad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

Double-Angle Identities:

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Power-Reducing Identities:

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$ $\quad \cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\quad \tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

ROUGH WORK FOR MULTIPLE-CHOICE

You may detach this page for convenient reference.
Exercise care when tearing along perforations.

ROUGH WORK FOR MULTIPLE-CHOICE