9.2 Permutations, $n \, \mathrm{P} \, r$

A permutation is the fundamental counting principle put into a formula.

Permutations (Rule A)

Permutations (Rule A) must follow the following three restrictions:

1. The n objects are all different

2. No object can be repeated

3. Order makes a difference (ex. xy is different from yx)

A permutation is the arrangement of r objects with $(r \leq n)$ can be written as $P(n, r) \; = \; n\mathrm{P}r$

$$n \, \mathrm{P} \, r = \frac{n!}{(n - r)!}$$

Note: Combinations and Permutations are similar but have a distinctive difference.

Permutations – order matters

Combinations – order does not matter

For example, **permutation** considers 538-2783 as being **different** from 583-2783, while **combinations** recognize the two as the **same**.

Another example, consider three people, Chris, Daryl, and Emma.

In a three-person committee, **combinations** considers these three people as one combination.

For a three-person executive committee with a president, vice-president, and treasurer, **permutations** would consider these people as $(3 \, \mathrm{P} \, 3 = 3!)$ 6 permutations.

Ex. How many different 3-digit numbers can be made using the numbers 1 through 7 by only using each number once?

$$7P3$$

$$= \frac{7!}{(7-3)!}$$

$$= \frac{7 \times 6 \times 5 \times 4!}{4!}$$

$$= 7 \times 6 \times 5 = 210$$

There are 210 permutations.

Ex. How many ways can a president, vice president and treasurer be selected from a class of 25 students?

$$25P3$$

$$= \frac{25!}{22!}$$

$$= 25 \times 24 \times 23$$

$$= 13800 \text{ ways}$$

Ex. 8 friends are going to watch a movie and want to sit together. In a row with 8 seats, how many ways can they pick their seats?

$$8\ P\ 8$$

$$= 8!$$

$$= 40320$$

Ex. 8 friends are going to dinner and sit an 8-person round table. How many ways can they sit together?

Because the table is round, the first person only has 1 choice (to sit in an open seat). The second person has 7 choices to sit relative to the first person. The third person has 6 choices relative to the first person... $7 P 7$

$$= 7!$$

$$= 5040$$

Ex. 8 friends are going to watch a movie and want to sit together. One couple always have to sit together. In a row with 8 seats, how many ways can they pick their seats?

Treat the couple as one person, so it's like 7 people. But the couple have 2 sitting arrangements.

$$2 P 2 \cdot 7 P 7$$

$$= 2! \cdot 7!$$

$$= 10080$$

Ex. 8 friends are going to watch a movie and want to sit together. The couple from the previous question broke up and cannot sit together. In a row with 8 seats, how many ways can they pick their seats?

The total ways of 8 people sitting is $8! = 40320$

The total ways the couple sit together is $2! \cdot 7! = 10080$

The total ways the broke up couple avoid each other is the difference between the previous two sums $= 40320 - 10080 = 30240$

Permutations (Rule B)

n objects where there are some objects that are the same

Comparing Permutations Rule A and B

Rule A: $abcd$ Rule B: $aabc$

Permutations (Rule B)
In the example above, $aabc$, we could consider the same objects to be different, so a_1a_2bc. So, the arrangement a_1a_2bc looks different from a_2a_1bc, but the two a's indistinguishable. Thus, the two arrangements are considered the same.

In order to not count the repeats, we divide the total possible arrangements by the factorial of each repeat.

So, arrangements for $aabc$ would be $\frac{4!}{2!} = 12$

Ex.   How many different 5-letter "words" can be formed from the letters "SWEET"?

E occurs twice, need to divide the possible outcomes by 2!

$\frac{5!}{2!} = 60$

Ex.   How many different words can be formed from BANANA using all the letters?

There are multiple letters that repeat. A occurs 3 times, while N occurs twice. Need to divide the total by 3! and 2!.

$\frac{6!}{3!2!} = 60$

Ex. How many different words can be formed from MISSISSIPPI?

        11 total letters, I occurs 4 times, S occurs 4 times, while P occurs twice.

$$\frac{11!}{4!4!2!} = 34650$$

Ex. Find the number of different ways of placing 16 balls in a row given that 4 are black, 3 are white, 7 are red and 2 are blue.

$$\frac{16!}{4!3!7!2!} = 14414400$$

9.2 Homework
1-21 odd