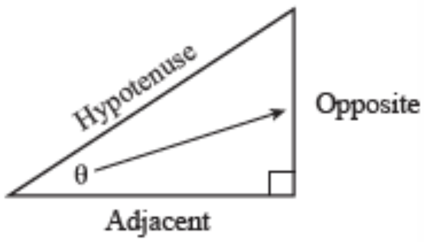
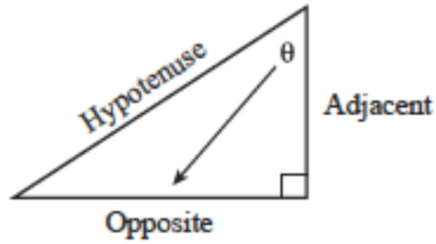
Ch 8 – Trigonometry

8.1 Sine, Cosine, and Tangent Ratios for Right Triangles





The names of the sides (opposite, adjacent) of the right triangles are relative to the angle θ

Trigonometry is the study of the ratio of the sides of a right triangle

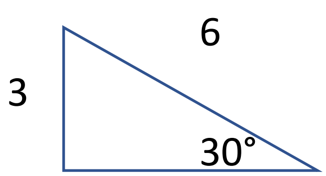
Ratio – comparison; can be written as a fraction

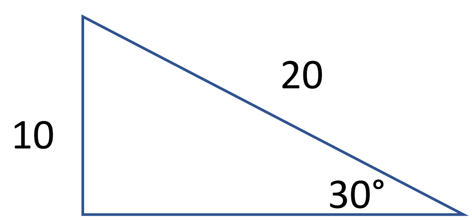
Ex 2:3 = $\frac{2}{3}$

Sine Ratio – the comparison of the opposite length to the hypotenuse length

$$\sin\theta = \frac{opp}{hyp}$$

Ex. $\sin 30° = 0.5$ or $\frac{1}{2}$

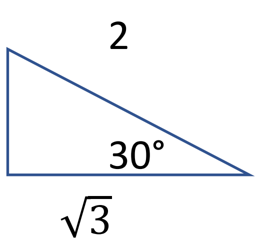


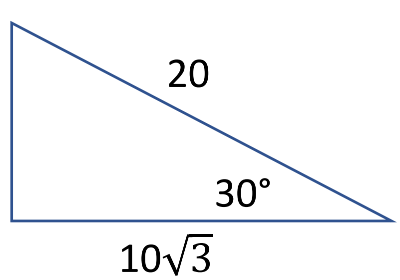


Cosine Ratio - the comparison of the adjacent length to the hypotenuse length

$$\cos\theta = \frac{adj}{hyp}$$

Ex. $\cos 30° = 0.866$ or $\frac{\sqrt{3}}{2}$ (exact value)





Tangent Ratio - the comparison of the opposite length to the adjacent length

$$\tan\theta = \frac{opp}{adj}$$

In summary,

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

SOH CAH TOA

Ex. Evaluate $\sin 22°$ to 4 decimal places

$= 0.3746$

For a right triangle with an angle of $22°$; from the perspective of the 22°, the opposite length divided by the hypotenuse length is 0.3746

Ex. Evaluate tan $80°$ to 4 decimal places

$= 5.6713$

Ex. Solve for θ to 1 decimal place, when $\cos\theta = 0.234$

Isolate θ

To "undo" the cosine operation, the reverse operation of cos is

$\cos^{-1}(\quad)$ → read as "arc cos"

Commonly read as inverse cos

$\theta = \cos^{-1}(0.234)$ shift cos 0.234 2nd cos 0.234

$\theta = 76.5°$

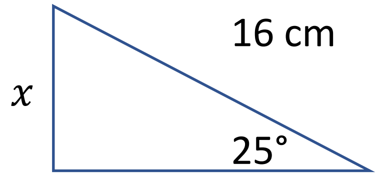
Ex. Solve for θ to 1 decimal place, when tan $\theta = 2.154$

$\theta = \tan^{-1}(2.154)$

$\theta = 65.1°$

Ex. Solve for the indicated measurement to 1 decimal place

a.



First, write a trig ratio that involves the given information

opp, hyp, θ → sine ratio

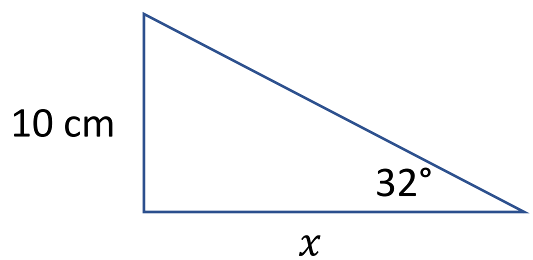
$$\sin 25° = \frac{x}{16}$$

Now, solve for x

$$x = 16 \sin 25°$$

$$x = 6.8 \text{ cm}$$

b.



opp, adj, θ → tangent ratio

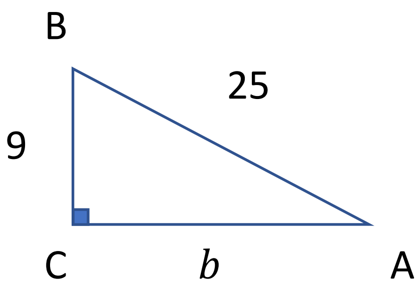
$\tan 32° = \frac{10}{x}$ to get rid of x from denominator, multiply both sides by x

$x \tan 32° = 10$ divide both sides by $\tan 32°$

$$x = \frac{10}{\tan 32°}$$

$$x = 16.0 \text{ cm}$$

Ex. Solve the right triangle. Round measurements to 1 decimal place.



When asked to solve the triangle, determine all measurements that are not given.
Need to determine $b, \angle A$ and $\angle B$

$$b^2 + 9^2 = 25^2$$
$$b^2 + 81 = 625$$
$$b^2 = 544$$
$b = \pm\sqrt{544} = \pm 4\sqrt{34}$ reject the negative

$b = 23.3$ units

$$\sin A = \frac{9}{25}$$
$$A = \sin^{-1}\left(\frac{9}{25}\right)$$
$$A = 21.1°$$

$$B + 21.1 + 90 = 180$$
$$B = 68.9°$$

8.1 Homework

1-2 bcf…, 3-4 bcf

8.2 Relationships Between Sine, Cosine, and Tangent

Verify that the following is true:

$\sin 0 = \cos 90$

$\sin 1 = \cos 89$

$\sin 2 = \cos 88$

…

$\sin 90 = \cos 0$

A pattern / relationship emerges.

To summarize this relationship between sine and cosine

Co-Function Identities

→ $\sin\theta = \cos(90° - \theta)$ or $\cos\theta = \sin(90° - \theta)$

This is an example of a Trigonometric Identity.

Ex. If $\sin 35° = \cos x$, determine the acute angle x.

Since $\sin\theta = \cos(90° - \theta)$, then:

$$\begin{aligned} \sin 35° &= \cos(90° - 35°) \\ &= \cos(55°) \end{aligned}$$

$\therefore x = 55°$

Here is the **Quotient Identity**:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Ex. Simplify the following $\frac{\sin 40°}{\sin 50°}$.

$$\begin{aligned} \sin 50° &= \cos(90° - 50°) \\ &= \cos 40° \end{aligned}$$

$$\frac{\sin 40°}{\sin 50°} = \frac{\sin 40°}{\cos 40°} = \tan 40°$$

Reciprocal Identities

$\sec\theta = \frac{1}{\cos\theta}$ secant

$\csc\theta = \frac{1}{\sin\theta}$ cosecant

$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$ cotangent

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$ → since $\sec\theta = \frac{1}{\cos\theta}$

$\tan^2\theta + 1 = \sec^2\theta$

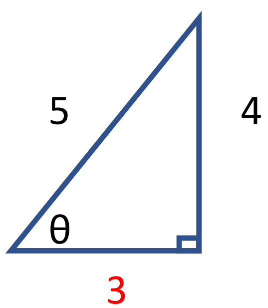
$1 + \cot^2\theta = \csc^2\theta$

$\sec\theta$ is read as **secant** of theta

$\csc\theta$ is read as **cosecant** of theta

$\cot\theta$ is read as **cotangent** of theta

Ex. Given $\sin\theta = \frac{4}{5}$, where $0° \leq \theta < 90°$, determine $\cos\theta$ and $\tan\theta$.



Use Pythagorean theorem to determine missing side which is equal to 3

$\cos\theta = \frac{3}{5}$ $\tan\theta = \frac{4}{3}$

Alternatively, the solution can be found using the Trig Identities

$\sin^2\theta + \cos^2\theta = 1$

Note: $\sin^2\theta = (\sin\theta)^2$ $\cos^2\theta = (\cos\theta)^2$ $\tan^2\theta = (\tan\theta)^2$

$\left(\frac{4}{5}\right)^2 + \cos^2\theta = 1$

$\frac{16}{25} + \cos^2\theta = 1$

$\cos^2\theta = \frac{9}{25}$

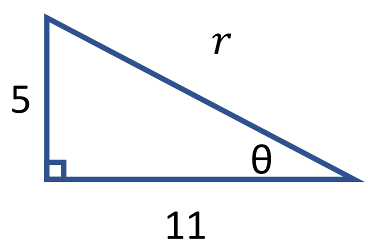
$\cos\theta = \frac{3}{5}, -\frac{3}{5}$ reject the negative

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$

Ex. Given $\tan\theta = \frac{5}{11}$, where $0° \leq \theta < 90°$

Determine $\cos\theta$ and $\sin\theta$, exact solutions only.



$r^2 = 5^2 + 11^2$

$r^2 = 25 + 121$

$r^2 = 146$

$r = \sqrt{146}, -\sqrt{146}$ reject the negative

$\cos\theta = \frac{11}{\sqrt{146}}$ $\sin\theta = \frac{5}{\sqrt{146}}$

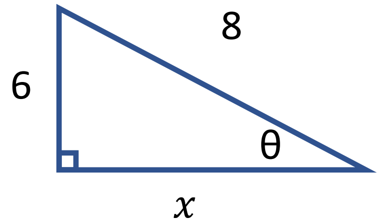
Answers can have radical denominators

→ but you can **rationalize the denominator**

$\frac{11}{\sqrt{146}} = \frac{11}{\sqrt{146}} \times \frac{\sqrt{146}}{\sqrt{146}} = \frac{11\sqrt{146}}{146}$ $\frac{5}{\sqrt{146}} = \frac{5}{\sqrt{146}} \times \frac{\sqrt{146}}{\sqrt{146}} = \frac{5\sqrt{146}}{146}$

$\therefore \cos\theta = \frac{11\sqrt{146}}{146}$ $\sin\theta = \frac{5\sqrt{146}}{146}$

Ex. Determine the 3 basic trig ratios ($\sin\theta$, $\cos\theta$, and $\tan\theta$). Exact value only!



$x^2 + 6^2 = 8^2$

$x^2 + 36 = 64$

$x^2 = 28$

$x = \pm\sqrt{28}$

$x = \pm 2\sqrt{7}$ reject the negative answer

$x = 2\sqrt{7}$

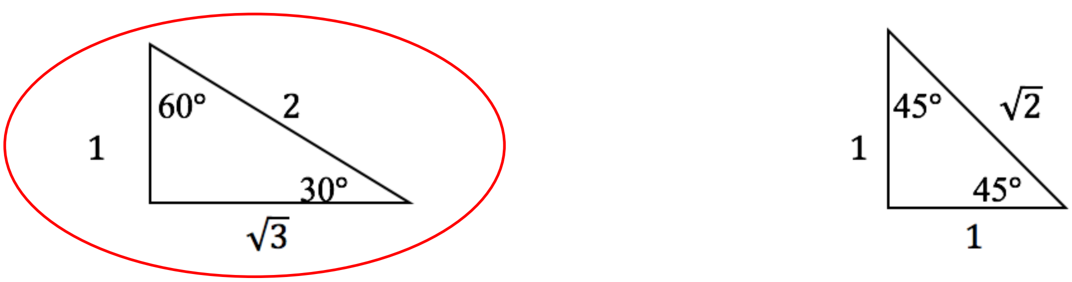
$\sin\theta = \frac{6}{8} = \frac{3}{4}$ $\cos\theta = \frac{2\sqrt{7}}{8} = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{6}{2\sqrt{7}} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

Ex. Evaluate without a calculator: $\tan^2 30° - \frac{1}{\cos^2 30°}$

Remember the special triangles

Special Triangles:



Since $\tan 30° = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ and $\cos 30 = \frac{\sqrt{3}}{2}$

$= \left(\frac{1}{\sqrt{3}}\right)^2 - \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1}{3} - \frac{1}{\frac{3}{4}}$

$$= \frac{1}{3} - \frac{4}{3}$$
$$= -1$$

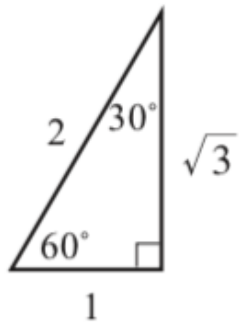
8.2 Homework:

#1-3 bcf…, 4, 5bc, 6bcf

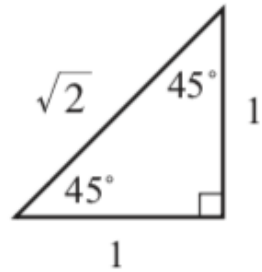
8.3 Special Angles

There are some well-known ratios that are associated with two special triangles. All answers in the section will be given as an exact answer; no approximate values.

The two Special Triangles you will need to know are:



30-60-90 Triangle



45-45-90 Triangle

Ratios Involving 30°

$\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\tan 30° = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$

Ratios Involving 60°

$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

Ratios Involving 45°

$\sin 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ $\cos 45° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ $\tan 45° = \frac{1}{1} = 1$

Summary of the information above:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Ex. Evaluate the expression $\frac{4\tan^3 2\theta}{5}$, given that $\theta = 30°$

Note: $\tan^2 x = (\tan x)^2$ and $\tan^3 x = (\tan x)^3$

$$= \frac{4\tan^3(2(30°))}{5}$$
$$= \frac{4\tan^3(60°)}{5}$$
$$= \frac{4(\tan 60°)^3}{5}$$
$$= \frac{4(\sqrt{3})^3}{5}$$
$$= \frac{4(\sqrt{27})}{5}$$
$$= \frac{4(3\sqrt{3})}{5}$$
$$= \frac{12\sqrt{3}}{5}$$

Ex. Evaluate the expression $\cos(3\theta - 75°)$, given that $\theta = 45°$

$$\cos(3\theta - 75°)$$
$$= \cos(3(45°) - 75°)$$
$$= \cos(135° - 75°)$$
$$= \cos 60°$$
$$= \frac{1}{2}$$

Ex. Find the exact value of the expression $\frac{2\tan 60°}{1-\tan^2 60°}$.

$$\frac{2\tan 60°}{1-\tan^2 60}$$
$$= \frac{2(\sqrt{3})}{1-(\sqrt{3})^2}$$
$$= \frac{2\sqrt{3}}{1-3}$$
$$= \frac{2\sqrt{3}}{-2}$$
$$= -\sqrt{3}$$

Ex. Evaluate the expression $\frac{2}{3\sin^2\frac{\theta}{2}}$, given that $\theta = 60°$

$$\frac{2}{3\sin^2\frac{\theta}{2}}$$
$$= \frac{2}{3\sin^2\left(\frac{60°}{2}\right)}$$
$$= \frac{2}{3\sin^2(30°)}$$
$$= \frac{2}{3\left(\frac{1}{2}\right)^2}$$
$$= \frac{2}{3\left(\frac{1}{4}\right)}$$
$$= \frac{2}{\frac{3}{4}}$$
$$= \frac{8}{3}$$

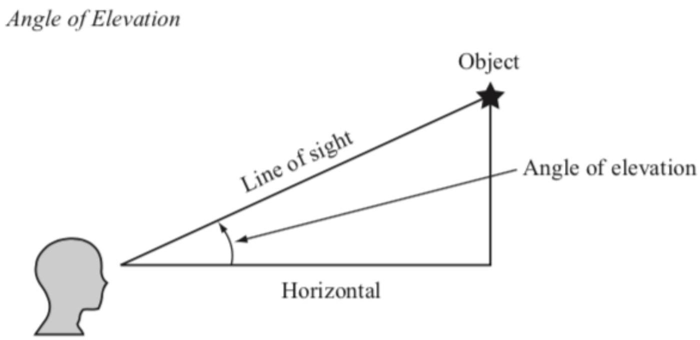
8.3 Homework

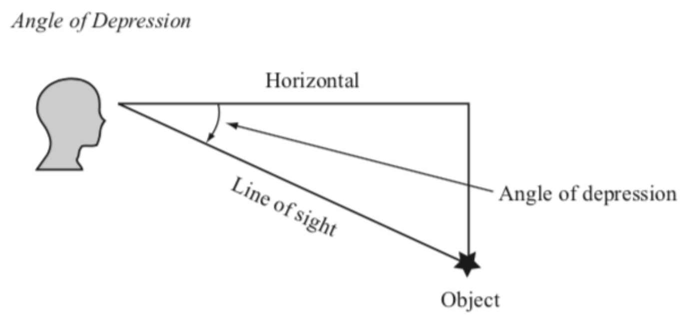
1-4 bcf…

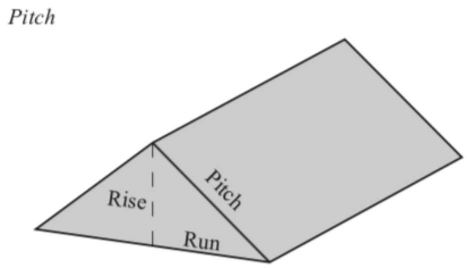
4j $\frac{\tan^2 30°}{2} - 3\sin^2 60° + \frac{4\cos^2 45°}{3}$

$$= \frac{\left(\frac{\sqrt{3}}{3}\right)^2}{2} - 3\left(\frac{\sqrt{3}}{2}\right)^2 + \frac{4\left(\frac{\sqrt{2}}{2}\right)^2}{3}$$
$$= \frac{\frac{3}{9}}{2} - 3\left(\frac{3}{4}\right) + \frac{4\left(\frac{2}{4}\right)}{3}$$
$$= \frac{1}{6} - \frac{9}{4} + \frac{2}{3}$$
$$= \frac{2}{12} - \frac{27}{12} + \frac{8}{12}$$
$$= -\frac{17}{12}$$

8.4 Applications of Trigonometry





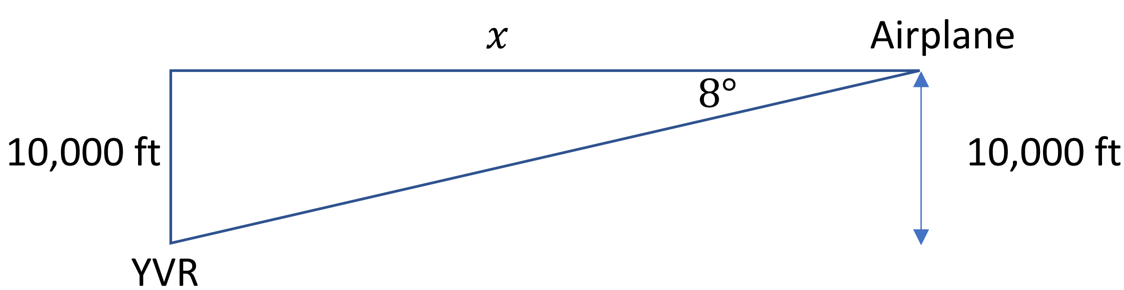


$\text{Pitch} = \frac{\text{Rise}}{\text{Run}}$

$\text{Pitch} = \text{Rise} : \text{Run}$

Ex. A pilot is required to approach Vancouver airport at an 8° angle of descent. If the plane is travelling at an altitude of 10 000 ft, at what horizontal distance from the airport should the descent begin?

8° angle of descent = 8° of angle of depression



Now write trig ratio

$\tan 8° = \frac{10000}{x}$

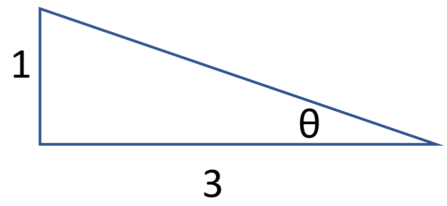
$x = \frac{10000}{\tan 8°}$

$x = 71153.697 \approx 71154 \text{ ft}$

Ex. A carpenter says that the pitch needed for rain to run properly off a roof is at least 4 to 12. Find the angle the roof makes with the horizontal.

pitch = slope

$4 \text{ to } 12 = 4:12 = \frac{4}{12} = \frac{1}{3}$



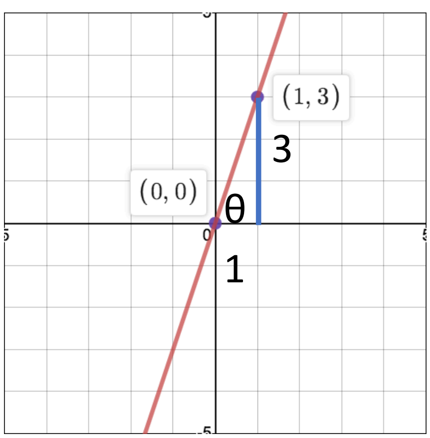
$\tan\theta = \frac{1}{3}$

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\theta = 18.4349 \approx 18.4°$

Ex. What is the acute angle between the line $y = 3x$ and the x −axis?

Draw $y = 3x$ on a cartesian plane.

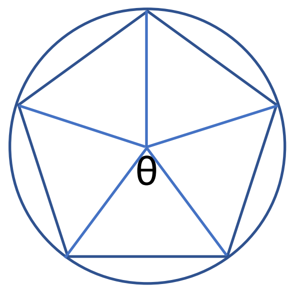


$\tan\theta = \frac{3}{1}$

$\theta = \tan^{-1}(3)$

$\theta = 71.5651 \approx 71.6°$

Ex. A regular pentagon is inscribed in a circle of radius 12 cm. Calculate the perimeter of the pentagon.

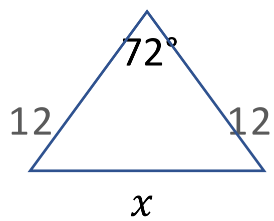


Inside pentagon, there are 5 triangles

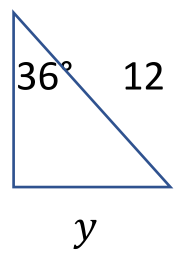
$\therefore 5\theta = 360$

$\theta = 72°$

radius = 12 cm



→



$x = 2y \qquad y = \frac{1}{2}x$

$\sin 36° = \frac{y}{12}$

$y = 12 \sin 36°$

$x = 2(12 \sin 36°)$

$x = 24 \sin 36°$

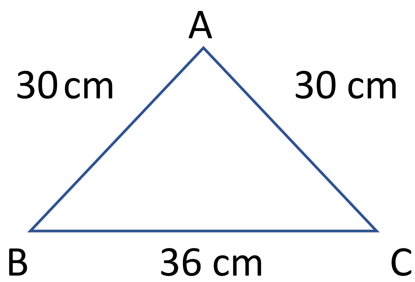
$x = 14.1$ cm

$P = 5(x)$

$P = 5(24 \sin 36°)$

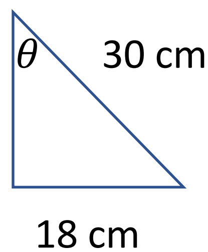
$P \approx 70.5$ cm

Ex. The equal sides of an isosceles triangle are 30 cm, and the third is 36 cm. Determine the measure of the interior angles of the triangle.



$A + B + C = 180$

$B = C$ (isosceles triangle)



$A = 2\theta$

$\sin\theta = \frac{18}{30}$

$\theta = \sin^{-1}\left(\frac{18}{30}\right)$

$\theta = 36.9°$

$A = 2\theta = 73.8°$

Since B and C are equal, $B + C = B + B = 2B$

$73.8° + 2B = 180$

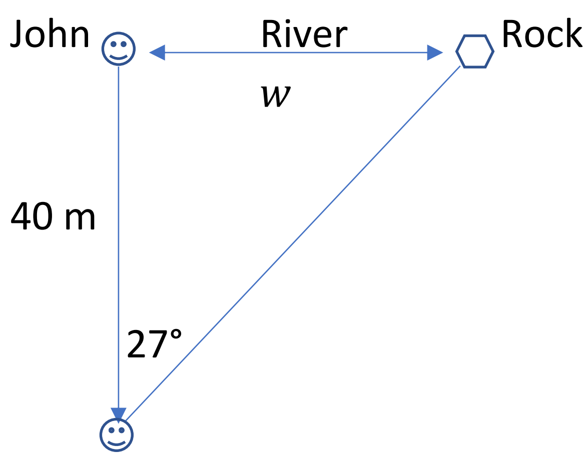
$2B = 106.2°$

$B = C = 53.1°$

The interior angles of the triangle are 53.1° , 53.1° , and 73.8°

8.4 Homework #1-16 all

Ex. #12 from 8.4



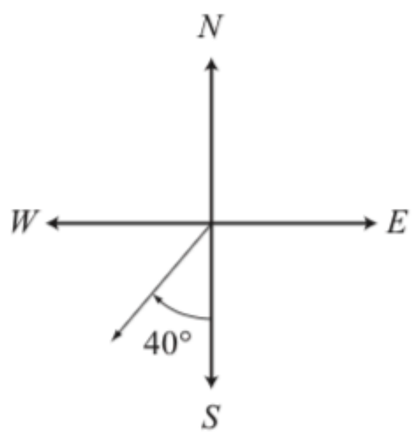
$\tan 27° = \frac{w}{40}$

$w = 40\tan 27°$

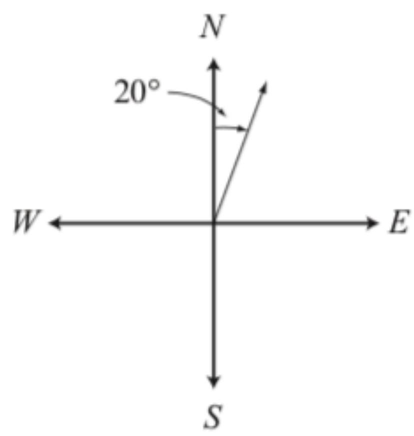
$w = 20.4$ m

8.5 Compound Trigonometry Applications

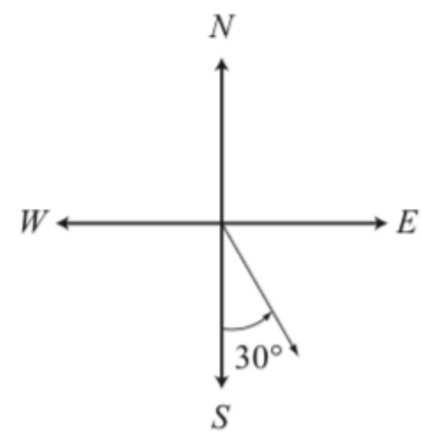
Bearings are used in navigation as a number that represents a direction of travel. A bearing measures the acute angle displacement, and direction, from a fixed north-south line.



$S\ 40^\circ\ W$
(40° west of south)



$N\ 20^\circ\ E$
(20° east of north)

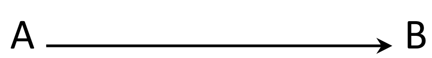


$S\ 30^\circ\ E$
(30° east of south)

Ex. A ship leaves port at 9 am and heads due east at 20 knots. At 11 am, to avoid a storm, the ship changes course to $N\ 50^\circ\ E$ (50° east of north). Find the ships' bearing and the distance from port at noon.

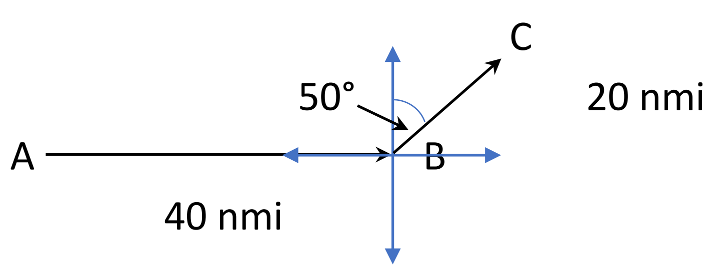
Note: $N\ 50^\circ\ E$ is read as 50° east of North

The ship travels due east for 2hrs at 20 knots; 40 nmi

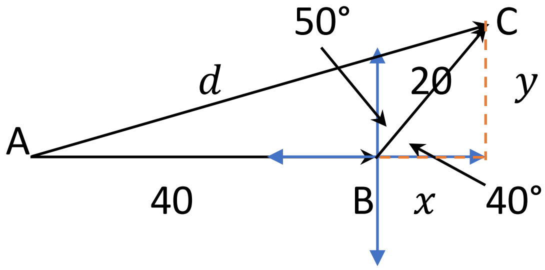


20 knots x 2 hrs = 40 nautical miles = 40 nmi

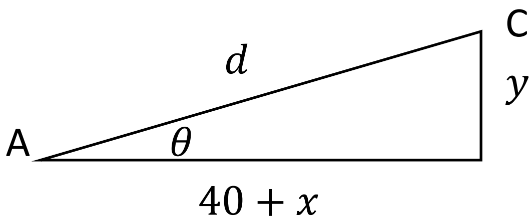
The ship travels $N\ 50^\circ\ E$ for 1 hr at 20 knots; 20 nautical miles = 20 nmi

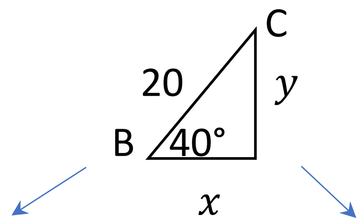


d is the distance from the ship at noon from the port



$90° - 50° = 40°$



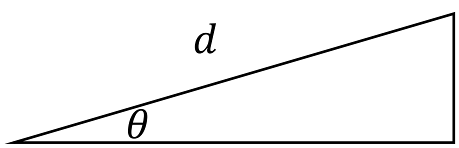


$\cos 40 = \frac{x}{20}$

$x = 20\cos 40$

$\sin 40 = \frac{y}{20}$

$y = 20\sin 40$



$20\sin 40 = 12.85575219$

$40 + 20\cos 40$

$= 55.32088886$

Solve for θ

$\tan\theta = \frac{20\sin 40}{40+20\cos 40}$

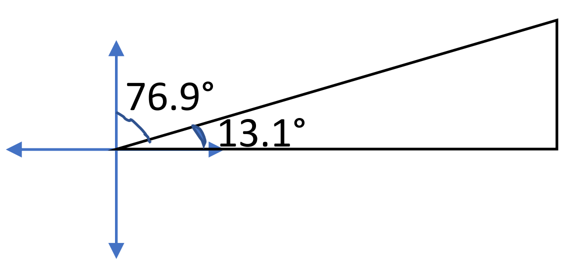
$\theta = \tan^{-1}\left(\frac{20\sin 40}{40+20\cos 40}\right)$

$\theta = 13.1°$

$\tan\theta = \frac{12.85575219}{55.32088886}$

$\theta = \tan^{-1}\left(\frac{12.85575219}{55.32088886}\right)$

$\theta = 13.1°$



$90° - 13.1° = 76.9°$

The bearing is 76.9° east of north

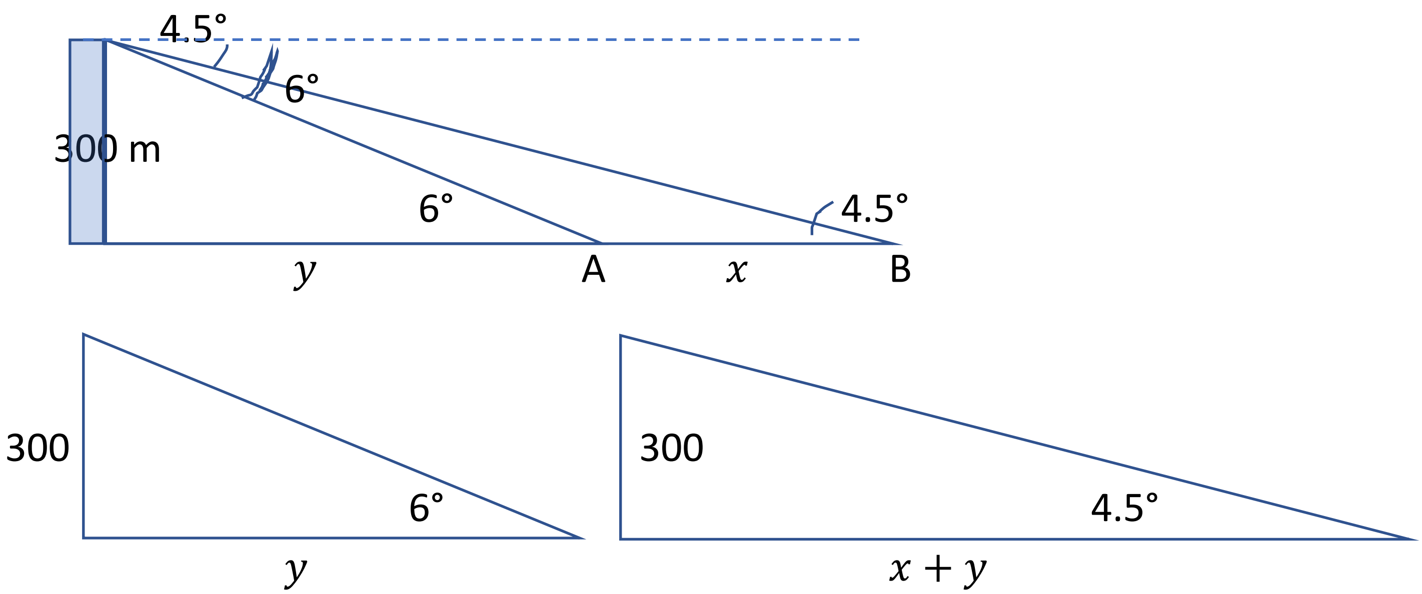
$d^2 = (20 \sin 40)^2 + (40 + 20 \cos 40)^2$

$d = \pm\sqrt{(20 \sin 40)^2 + (40 + 20 \cos 40)^2}$, reject the negative

$d = 56.8$ nmi

∴ The bearing is $N\ 76.9°\ E$ (76.9° east of north) at 56.8 nmi from port.

Ex. A lighthouse keeper 300 m above sea level spots two ships directly off shore. The angles of depression of the ships are 4.5° and 6°. Determine how far are the ships apart from each other.



$\tan 6 = \frac{300}{y}$

$y = \frac{300}{\tan 6}$

$\tan 4.5 = \frac{300}{x+y}$

$x + y = \frac{300}{\tan 4.5}$

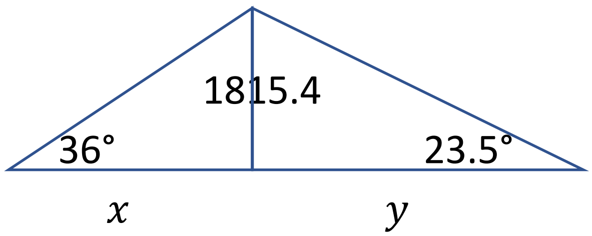
$x + \frac{300}{\tan 6} = \frac{300}{\tan 4.5}$

$x = \frac{300}{\tan 4.5} - \frac{300}{\tan 6}$

$x = 957.6$ m

∴ the distance between the ships is 957.6 m

Ex. The CN Tower is 1815.4 ft high. An observer measures the angle of elevation to the top of the tower at 36°. Another person directly opposite the tower from the first person measures the angle of elevation to the top of the tower at 23.5°. Determine the distance between the two observers.



$\tan 36 = \frac{1815.4}{x}$ $\tan 23.5 = \frac{1815.4}{y}$

$x = \frac{1815.4}{\tan 36}$ $y = \frac{1815.4}{\tan 23.5}$

total distance $= x + y$

$= \frac{1815.4}{\tan 36} + \frac{1815.4}{\tan 23.5}$

$= 6673.8 \text{ ft}$

8.5 Homework:

2, 4, 6, 8, 10, 12