9.3 Combinations, ${}_n C_r$

Combinations are similar to permutations, where they both look for possible ways/outcomes to accomplish a task.

Combinations are arrangements of r number of objects chosen from n number of objects where:

- n objects are all different
- no objects can be repeated
- order does **NOT** make a difference (ab and ba are the same combination)

The number of combinations choosing r objects from n objects is given by:

$$ {}_n C_r = \frac{n!}{r!(n-r)!} \qquad \text{where } r \leq n $$

Other ways to write combinations:

$$ n\,C\,r \quad = \quad C(n,r) \quad = \quad \binom{n}{r} $$

Combination Examples

Ex. How many ways can 2 people be selected from a group of 6 people?

Order does not matter $\quad n = 6 \quad r = 2$

$$ {}_6 C_2 $$

$$ = \frac{6!}{2!(6-2)!} $$

$$ = \frac{6!}{2!4!} $$

$$ = \frac{720}{2 \times 24} = 15 $$

There are 15 ways to select 2 people from a group of 6.

Ex. How many 5 card hands are possible in a regular deck of 52 cards?

Order does not matter $n = 52$ $r = 5$

${}_{52}C_5$

$= \frac{52!}{5!47!}$

$= \frac{52 \times 51 \times 50 \times 49 \times 48 \times 47!}{5!47!}$

$= 2{,}598{,}960$

There are $2{,}598{,}960$ different possible hands.

Ex. How many different tickets are possible when playing Lotto 6/49? (must pick 6 numbers out of 49 numbers in any order)

${}_{49}C_6$

$= \frac{49!}{6!43!}$

$= \frac{49 \times 48 \times 47 \times 46 \times 45 \times 44 \times 43!}{6!43!}$

= 13,983,816

There are 13,983,816 possible combinations

Ex. From 6 students and 4 teachers, a committee of 2 students and 2 teachers must be chosen. How many ways can this be done?

(number of ways to choose students)(number of ways to choose teachers)

${}_6C_2 \times {}_4C_2$

= 15 x 6 = 90

There are 90 possible ways

Ex. If night school offers 100 courses, 8 of which are in Mathematics, and you select 4 courses by random selection, how many possibilities include one Mathematics course?

${}_8C_1 \times {}_{92}C_3$

$$\binom{8}{1}\binom{92}{3} = 1{,}004{,}640$$

There are $1{,}004{,}640$ possibilities.

Solving Combination Equations

Ex. Solve for n. ${}_nC_2 = 66$

$$\frac{n!}{2!(n-2)!} = 66$$

$$\frac{n(n-1)(n-2)!}{2(n-2)!} = 66$$

$$n^2 - n = 132$$

$$n^2 - n - 132 = 0$$

$$(n-12)(n+11) = 0$$

$$n = 12, -11$$

$$n = 12$$

9.3 Homework

2, 5-7, 9, 11