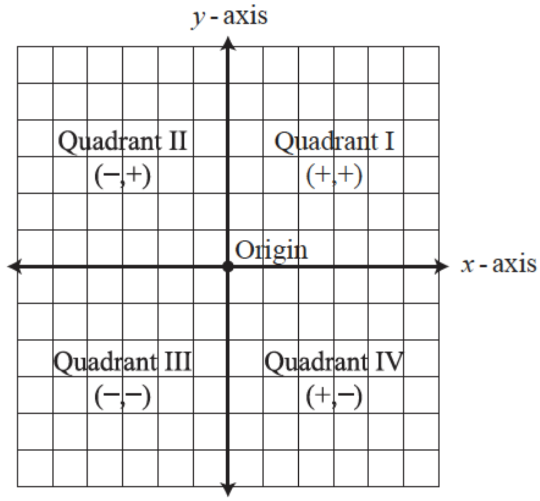
Chapter 3 – Relations and Functions

3.1 – Relations

Coordinate System

Cartesian Plane

Coordinate: (x, y)



Ex. Which Quadrant does the coordinate belong to?

A (-4, 5)

Q2 or QII

B (-3, -7)

Q3 or QIII

C (5, 0)

None

D (5, 3)

Q1 or QI

E (8, -2)

Q4 or QIV

Relations

A connection / relationship between two sets of information

When one part of the relationship depends on the other, we classify them as being dependent and independent.

Domain and Range

Domain = possible independent values
Range = possible dependent values

Ex. Relation: Vehicles and their number of wheels

S = { (sedan, 4), (motorcycle, 2), (bus, 6), (semi, 10), (truck, 4) }

For the relation S, determine the domain and range.

Domain
D: { bus, motorcycle, sedan, semi, truck}

Range
R: { 2, 4, 6, 10 }

Ex. Vehicle vs. Wheels relation

For the relation above, determine the independent and dependent variable.

Since the number of wheels depends on the type of vehicle, the independent variable is the vehicle type while the dependent variable is the number of wheels.

Independent variable = vehicle type

Dependent variable = number of wheels

Ex. For the relation $y = 2x$, determine the independent and dependent variable.

Since the value of y is equal to 2 times the value of x, we say that the value of y depends on the value of x.

Independent variable = x

Dependent variable = y

Discrete vs Continuous

Discrete: a finite number of possible values

Continuous: an infinite number of possible values

Ex. For $S = \{(0,0), (1,2), (2,4), (3,6)\}$, write the domain and range.

$D = \{0, 1, 2, 3\}$

$R = \{0, 2, 4, 6\}$

The domain and range are both **discrete.**

Ex. For the linear relation $y = 2x$, write the domain and range.

Since there are no limits to the x and y values

$D: \{x \in \mathbb{R}\}$ x is an ∈lement of the ℝeal number set

$R: \{y \in \mathbb{R}\}$ y is an ∈lement of the ℝeal number set

The domain and range are both **continuous**

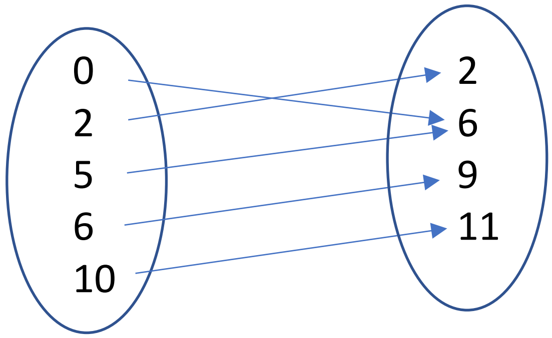
Ex. For the set of ordered pairs $S = \{(0,6), (2,2), (5,6), (6,9), (10,11)\}$, answer the questions below:

a. Write the domain and range

D: $\{0, 2, 5, 6, 10\}$

R: $\{2, 6, 9, 11\}$

b. Draw an arrow diagram to represent the relation S

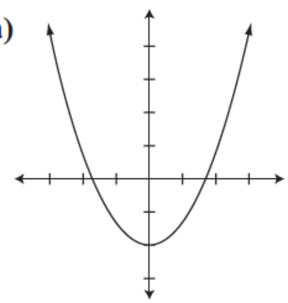


c. Write the relation S as a table of values

x	0	2	5	6	10
y	6	2	6	9	11

Ex. Write the domain and range for each of the following graphs.

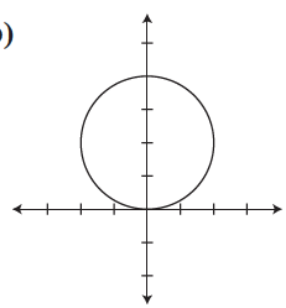
a)



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq -2\}$

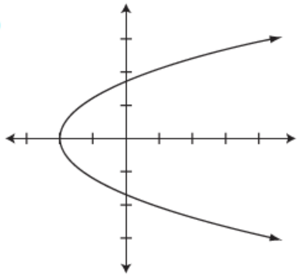
b)



D: $\{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$

R: $\{y \in \mathbb{R} \mid 0 \leq y \leq 4\}$

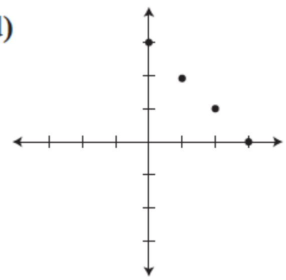
c)



D: $\{x \in \mathbb{R} \mid x \geq -2\}$

R: $\{y \in \mathbb{R}\}$

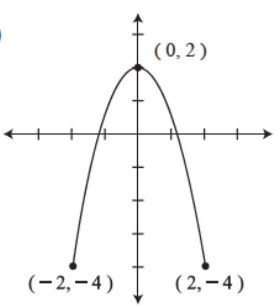
d)



D: $\{0, 1, 2, 3\}$

R: $\{0, 1, 2, 3\}$

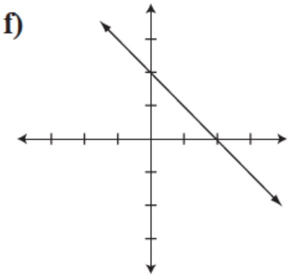
e)



D: $\{x \in \mathbb{R} \mid -2 \leq x \leq 2\}$

R: $\{y \in \mathbb{R} \mid -4 \leq y \leq 2\}$

f)



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$

Homework

3.1 # All

3.2 Functions

Function: a special relation between two sets of values
Every value in the domain (independent variable) has only 1 outcome in the range (dependent variable)

Ex. Relation: A child and its (biological) parents
Function or relation?

It's a relation because there should be 2 outcomes

Ex. Relation: A child and its (biological) mother
Function or relation?

It's a function because there should be only 1 outcome

One-To-One Function
Every input (independent variable) has its own unique output (dependent).

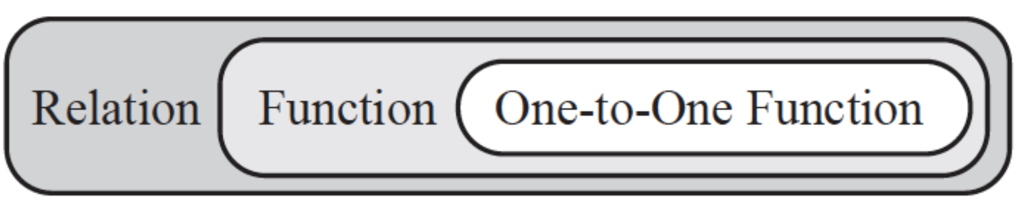
Ex. Function: A person and its first name
Function or One to One?

This is a function because multiple people can have the same first name

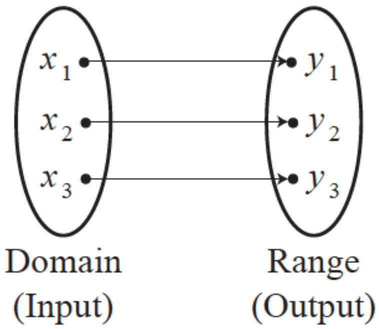
Ex. Function: A person and their DNA
Function or One to One?

This is a one-to-one function because everyone has unique DNA

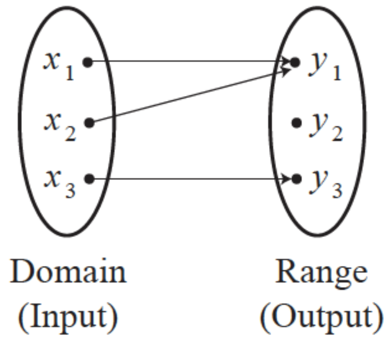
Hierarchy of Relations, Functions, and One-to-One Functions



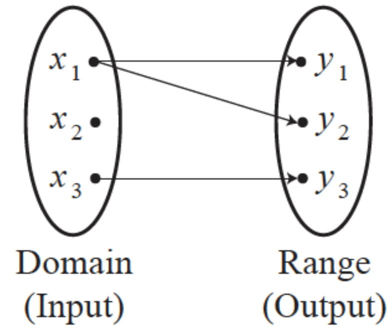
Arrow Diagram



A one-to-one function



A function, but not one-to-one. Both x_1 and x_2 go to y_1.



Not a function, just a relation. x_1 goes to both y_1 and y_2

Vertical Line Test (VLT) -> function or non-function?

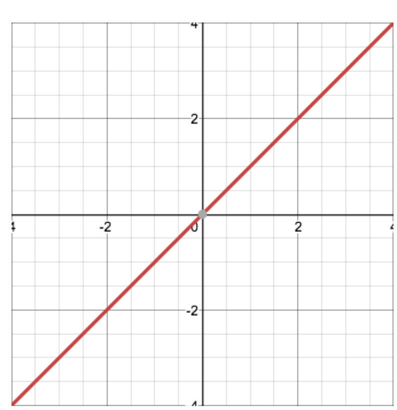
A graph is considered a function, if it passes the vertical line test. If a graph has multiple points that can be connected by a vertical line, the graph has failed the vertical line test; not a function.

Horizontal Line Test (HLT) -> one-to-one or function?

A function is considered one to one, if it passes the horizontal line test. If a graph has multiple points that can be connected by a horizontal line, the graph has failed the horizontal line test; not a one to one function.

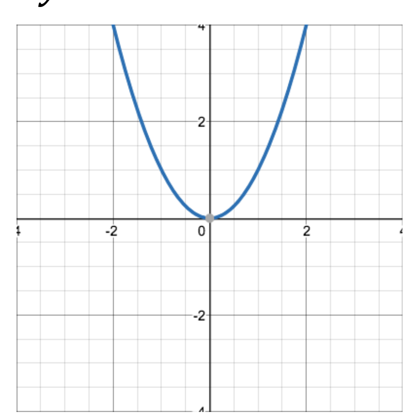
Ex. Relation, Function, or One-to-One?

a. $y = x$



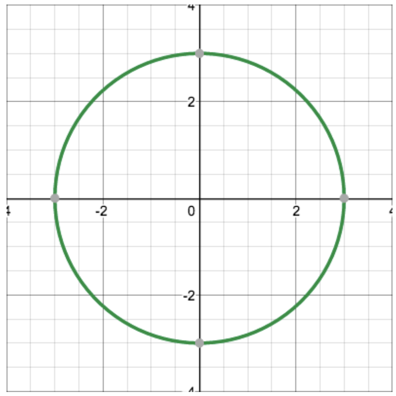
One to One

b. $y = x^2$



Function

c. $x^2 + y^2 = 9$



Relation

Function Notation $y = f(x)$

In function notation, the y in an equation is commonly replaced with $f(x)$; other possible replacements are allowed.

$f(x)$ read as "f of x" or "function of x"

$y = f(x)$ read as "y is a function of x"
"y equals to f of x"

Replacing y with $f(x)$

$y = 2x + 1$	Linear Equation (Relation)
$f(x) = 2x + 1$	Linear Function
$y = 2x^2 + 3x + 1$	Quadratic Equation
$f(x) = 2x^2 + 3x + 1$	Quadratic Function

Other Equations using Function Notation

$d = v_i t + \frac{1}{2}at^2$	distance equation from Physics
$d(t) = v_i t + \frac{1}{2}at^2$	distance function
$F = \frac{9}{5}c + 32$	Celsius to Fahrenheit conversion equation
$F(c) = \frac{9}{5}c + 32$	C to F function

Equation versus Function Notation

Linear Equation:

Ex. Given $y = 3x + 5$, find the value of y when x is 8.

$$y = 3x + 5$$
$$y = 3(8) + 5$$
$$y = 29$$

When $x = 8$ the value of $y = 29$

$\therefore (8, 29)$

Linear Function:

Ex. Given $f(x) = 3x + 5$, find $f(8)$.

$$f(x) = 3x + 5$$
$$f(8) = 3(8) + 5$$
$$f(8) = 29$$

Note: $f(8) = 29$ is equivalent to when $x = 8$, y equals to 29

3.2 Homework

1-3 bcf…, 4-6, 7bc, 8, 10, 13, 15, 16, 18

3.3 Linear Equations

Slope-Intercept Form of a Linear Relation

$y = mx + b$ slope-intercept form of a line

$\text{slope} = m$ $y - \text{intercept} = b$

$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ y-int: $(0, b)$

Standard Form of a Linear Relation

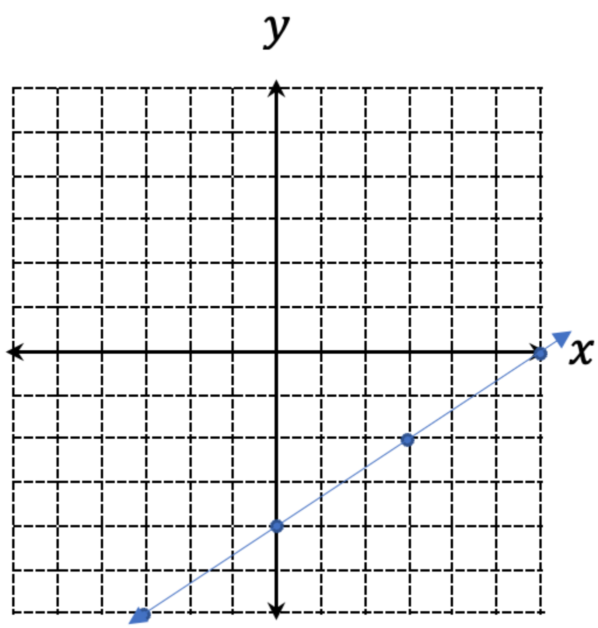
$\boldsymbol{Ax + By = C}$ standard form

A, B, C are $\mathbb{Z}$, and A must be positive
> this form is good for finding the x and y intercepts

Graphing Linear Relations in Slope-Intercept Form

Ex. Graph $y = \frac{2}{3}x - 4$

1. plot y-int
2. use slope to find additional points
3. connect the coordinates



$m = \frac{2}{3} = \frac{up\ 2}{right\ 3}$ or $\frac{-2}{-3} = \frac{down\ 2}{left\ 3}$

$b = -4 \quad \rightarrow \quad (0, -4)$

Graphing Linear Relations in Standard Form

Method 1: Use standard form equation to find x and y intercepts

Ex. Graph $2x + 4y = 12$

Find x intercept → make $y = 0$, solve for x

$2x + 4(0) = 12$

$2x = 12$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$ $\therefore (6, 0)$

Find y intercept → make $x = 0$, solve for y

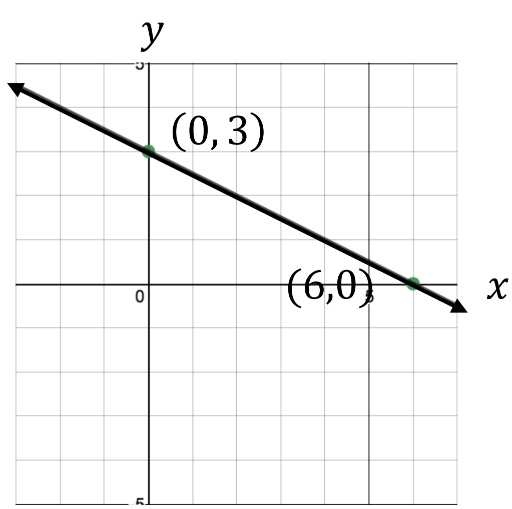
$2(0) + 4y = 12$

$4y = 12$

$\frac{4y}{4} = \frac{12}{4}$

$y = 3$ $\therefore (0, 3)$

Graph the 2 intercepts and connect the coordinates



Method 2: Convert $2x + 4y = 12$ to $y = mx + b$

Ex. Graph $2x + 4y = 12$

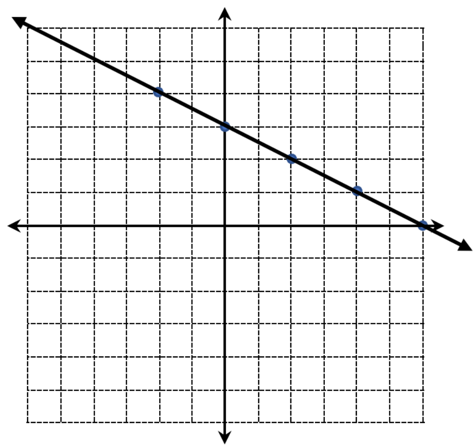
Convert standard form into $y = mx + b$ by isolating y

$$4y = -2x + 12$$

$$\frac{4y}{4} = -\frac{2x}{4} + \frac{12}{4}$$

$$y = -\frac{1}{2}x + 3 \quad \rightarrow \quad m = -\frac{1}{2} = \frac{-1}{2} = \frac{down\ 1}{right\ 2}$$

and $b = 3 \quad \rightarrow \quad (0, 3)$



Verifying if a Coordinate Belongs on a Line

If the values of the coordinate are substituted into the equation, a true statement would suggest that it is on the line, while a false statement would suggest it is NOT on the line.

Ex. For the linear equation $y = \frac{2}{3}x - 4$, determine if the following coordinates are on the line.

a. $(-3, -6)$

$$-6 = \frac{2}{3}(-3) - 4$$
$$-6 = -2 - 4$$
$$-6 = -6$$

Since this is **true**,
$(-3, -6)$ is on the line

b. $(0, 5)$

$$5 = \frac{2}{3}(0) - 4$$
$$5 = 0 - 4$$
$$5 = -4$$

Since this is **false**,
$(0, 5)$ is **NOT** on the line

Ex. For the linear equation $2x + 4y = 12$, determine if the following coordinates are on the line.

a. $(2, -2)$

$2(2) + 4(-2) = 12$
$4 - 8 = 12$
$-4 = 12$

Since this is **false**,
$(2, -2)$ is **NOT** on the line

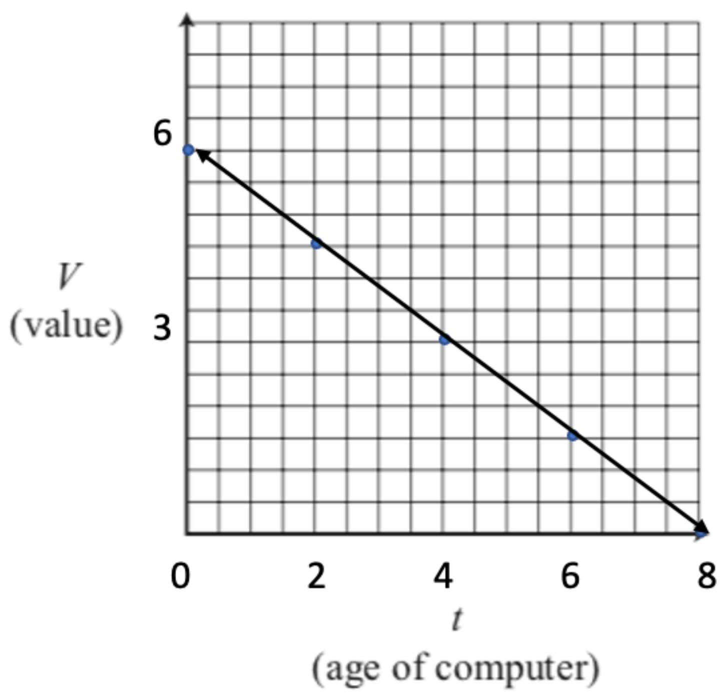
b. $(-8, 7)$

$2(-8) + 4(7) = 12$
$-16 + 28 = 12$
$12 = 12$

Since this is **true**,
$(-8, 7)$ is on the line

Ex. #5 from 3.3 on pg 136

The value V of a computer in hundreds of dollars is given by $V = -\frac{3}{4}t + 6$, where t is the number of years since the computer was bought. Graph the equation, and use the graph to estimate what the computer is worth after four years.



After 4 years, the computer is worth \$300

3.3 Homework
3 – 4 bcf…, 6, 7, 10, 11