Chapter 5 – Linear Equations

5.1 Different Forms of Linear Equations

Recall:

Slope-Intercept form	$y = mx + b$
General Form	$Ax + By + C = 0$
Standard Form	$Ax + By = C$
	A, B, C are integers and $A \geq 0$

Converting from standard to slope-intercept form, $\boldsymbol{y = mx + b}$

Isolate y in $Ax + By = C$.

Ex. Convert to slope intercept form, $3x - 5y = 25$.

> isolate the variable y

$$-5y = -3x + 25$$

$$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{25}{-5}$$

$$y = \frac{3}{5}x - 5$$

Converting from slope-intercept to standard form, $\boldsymbol{Ax + By = C}$

Put x and y on one side, and ensure all coefficients are integers, and A must be positive.

Ex. Convert to standard form, $y = \frac{2}{3}x + \frac{11}{4}$.

$$-\frac{2}{3}x + y = \frac{11}{4}$$

$$\left[-\frac{2}{3}x + y = \frac{11}{4}\right] \times -12$$ multiply both sides by -12

$$(-12) \times -\frac{2}{3}x + (-12) \times y = (-12) \times \frac{11}{4}$$

$$8x - 12y = -33$$

Find the equation of the linear function $(\boldsymbol{y} = \boldsymbol{mx} + \boldsymbol{b})$

Ex. A line has slope of $\frac{4}{3}$ and passes through $(5, 8)$

Option 1: using $y = mx + b$

$y = \frac{4}{3}x + b$ substitute in the slope

Sub (5, 8) into the equation, and solve for b

$8 = \frac{4}{3}(5) + b$

$8 = \frac{20}{3} + b$

$8 - \frac{20}{3} = b$

$b = \frac{4}{3}$

$\therefore y = \frac{4}{3}x + \frac{4}{3}$

Option 2: using $y - y_1 = m(x - x_1)$

$m = \frac{4}{3}$ $(5, 8)$

$y - 8 = \frac{4}{3}(x - 5)$ this is in point-slope form

By isolating the y, the equation will be in slope-intercept form

$y - 8 = \frac{4}{3}x - \frac{20}{3}$

$y = \frac{4}{3}x - \frac{20}{3} + 8$

$y = \frac{4}{3}x + \frac{4}{3}$

Ex. Find the equation of the line that passes through $(2, 4)$ and $(4, -5)$.

Find the slope first

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 4}{4 - 2}$$

$$= -\frac{9}{2}$$

Use point-slope form with $m = -\frac{9}{2}$ and $(2, 4)$

$$y - 4 = -\frac{9}{2}(x - 2)$$

$$y - 4 = -\frac{9}{2}x + 9$$

$$y = -\frac{9}{2}x + 9 + 4$$

$$y = -\frac{9}{2}x + 13$$

Homework

5.1 # 2 – 11 bcf…

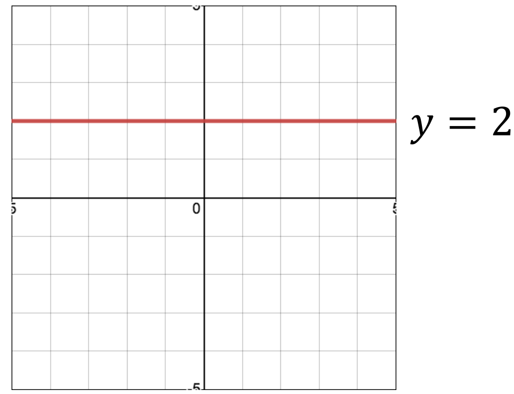
5.2 – Special Cases of Linear Equations

Horizontal Lines

Horizontal lines all have a slope equal to 0.

$\therefore$ for $y = mx + b$, where $m = 0$

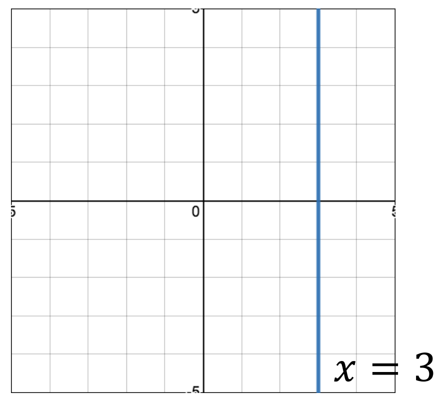
$$y = 0x + b$$
$$y = b$$



All horizontal line equations have the form:

$y = b$ $\qquad$ b is the y-intercept

Vertical Lines



Vertical lines all have an undefined slope (or equal to infinity $\pm\infty$).

$x = k$ $\qquad$ k is the x-intercept

Write the Equation of a line given two points, $y = mx + b$

Ex. $\quad (4, -11)$ and $(-5, 6)$

$$m = \frac{6-(-11)}{-5-4} = \frac{17}{-9} = -\frac{17}{9}$$

$$y - y_1 = -\frac{17}{9}(x - x_1)$$

$(-5, 6)$:

$$y - 6 = -\frac{17}{9}\left(x - (-5)\right)$$

$$y = -\frac{17}{9}x - \frac{85}{9} + 6$$

$$y = -\frac{17}{9}x - \frac{31}{9}$$

$(4, -11)$:

$$y + 11 = -\frac{17}{9}(x - 4)$$

$$y = -\frac{17}{9}x + \frac{68}{9} - 11$$

$$y = -\frac{17}{9}x - \frac{31}{9}$$

Parallel and Perpendicular Lines

When two lines are parallel, they all have the same slope

Ex. For $y_1 = 3x - 5$ and $y_2 = 3x + 1$, are the lines parallel, perpendicular, or neither?

$m_1 = 3 \qquad m_2 = 3$

$\therefore$ y_1 and y_2 are parallel

Ex. For $3x + 4y_1 = 5$ and $y_2 = -\frac{3}{4}x + 3$, are the lines parallel, perpendicular, or neither?

$m_1 = -\frac{3}{4} \qquad m_2 = -\frac{3}{4}$

$\therefore$ y_1 and y_2 are parallel

When two lines are perpendicular, their slopes are negative reciprocals of each other

Ex. For $y_1 = \frac{2}{3}x + 2$ and $y_2 = -\frac{3}{2}x - 9$, are the lines parallel, perpendicular, or neither?

$m_1 = \frac{2}{3} \qquad m_2 = -\frac{3}{2}$

If $m_1 \cdot m_2 = -1$, then the lines are perpendicular

$\frac{2}{3} \cdot -\frac{3}{2} = -1$

$\therefore$ y_1 and y_2 are perpendicular

Ex. For $4x - 5y = 6$ and $5x + 4y = 9$, are the lines parallel, perpendicular, or neither.

Option 1: convert both to $y = mx + b$

L_1: $4x - 5y = 6$

$-5y = -4x + 6$

$y = \frac{4}{5}x - \frac{6}{5}$

L_2: $5x + 4y = 9$

$4y = -5x + 9$

$y = -\frac{5}{4}x + \frac{9}{4}$

Option 2: determine the slope for each using $m = -\frac{A}{B}$

L_1: $4x - 5y = 6$

$m_1 = -\frac{4}{-5} = \frac{4}{5}$

L_2: $5x + 4y = 9$

$m_2 = -\frac{5}{4}$

Because the slopes are negative reciprocals of each other, the two lines are perpendicular $\perp$

If $m_1 \cdot m_2 = -1$, then the lines are perpendicular

$\frac{4}{5} \cdot -\frac{5}{4} = -1$

Ex. What is the slope of all ordered pairs of the form $\left(x, \frac{3}{2}x\right)$?

$\left(x, \frac{3}{2}x\right)$ same as (x, y)

So, $y = \frac{3}{2}x$

$\therefore m = \frac{3}{2}$

Ex. What is the slope of all ordered pairs of the form $\left(\frac{1}{2}x, \frac{3}{4}x\right)$?

$\left(\frac{1}{2}x, \frac{3}{4}x\right) = \left(\frac{1}{2}x, \frac{3}{2}\left(\frac{1}{2}x\right)\right)$ same as (x, y)

So, $y = \frac{3}{2}x$

$\therefore m = \frac{3}{2}$

or

Let $x = 0 \rightarrow (0, 0)$ $x = 4 \rightarrow (2, 3)$

Use slope equation to find the slope

$m = \frac{3-0}{2-0} = \frac{3}{2}$

5.2 Homework:

1-5, 6bcf…, 7bcf…, 8bcf…, 10, 11, 14, 15, 18

5.3 – Equations of Parallel and Perpendicular Lines

Ex. Write the equation of a line ($y = mx + b$) that is parallel to $7x + 5y = 35$ and passes through the point $A(21, 4)$.

$m = -\frac{A}{B} = -\frac{7}{5}$ use slope formula for standard form

$y - y_1 = m(x - x_1)$ point-slope form

$y - 4 = -\frac{7}{5}(x - 21)$

$y = -\frac{7}{5}x + \frac{147}{5} + 4$

$y = -\frac{7}{5}x + \frac{167}{5}$ slope-intercept form

Convert to standard form $Ax + By = C$

$\frac{7}{5}x + y = \frac{167}{5}$

Reminder: A, B, and C must be integers and $A \geq 0$

So, multiply both sides by the LCD, 5

$7x + 5y = 167$ standard form

Ex. Write the equation of a line (in $y = mx + b$) that is perpendicular to $6x - 3y = 1$ and passes through $(5, 2)$.

$m = -\frac{6}{-3} = 2$ $m_\perp = -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{1}{2}(x - 5)$ point-slope form

$y = -\frac{1}{2}x + \frac{9}{2}$ slope-intercept form

Convert to standard form

$\frac{1}{2}x + y = \frac{9}{2}$

$x + 2y = 9$ standard form

Note: Final answers must be in slope-intercept form or standard form

Ex. Find the equation of a line perpendicular to $2x - 3y = 7$ with the same y-intercept as $5x - 2y = 10$. Leave answer in both slope-intercept and standard form.

Find perpendicular slope, line is perpendicular to $2x - 3y = 7$

$m = -\frac{A}{B} = -\frac{2}{-3} = \frac{2}{3}$

$m_{\perp} = -\frac{3}{2}$

y-intercept:

Same y-intercept as $5x - 2y = 10$

Recall, y-intercept occurs when $x = 0$

$5(0) - 2y = 10$

$-2y = 10$

$y = -5$ y-intercept

$\therefore y = -\frac{3}{2}x - 5$ slope-intercept form

Convert to standard form

$\frac{3}{2}x + y = -5$

$3x + 2y = -10$ standard form

5.3 Homework

1 – 5 bcf…, 7, 8, 11, 12

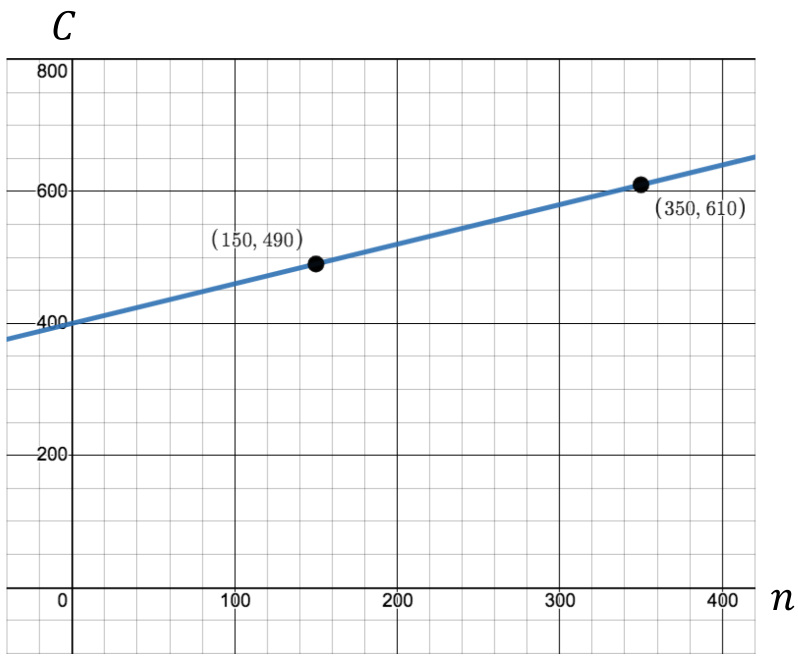
5.4 – Linear Applications and Modelling

Ex. It costs a popcorn vendor \$490 to make 150 bags of popcorn, and \$610 to make 350 bags.

a. Graph the linear relation

Cost depends on the number of bags
∴ cost is dependent and number of bags is independent

Two coordinates are: $(150, 490)$ and $(350, 610)$



b. Find the cost equation

Let C = cost and n = number of bags

$$m = \frac{610-490}{350-150} = \frac{120}{200} = \frac{3}{5} \quad \text{or} \quad m = 0.6$$

∴ it costs \$0.60 per bag (this is rate)

$$C - 490 = 0.6(n - 150)$$

$$C = 0.6n - 90 + 490$$

$$C = 0.6n + 400$$

c. Determine the fixed cost.

The fixed cost is the cost when you make 0 bags

$$C = 0.6(0) + 400$$

$$C = 400$$

The fixed cost is \$400

d. Determine the cost of making 800 bags of popcorn

$$C = 0.6(800) + 400$$

$$C = 480 + 400$$

$$C = 880$$

It costs \$880 to make 800 bags.

e. How much would you have to charge per bag to break even when making 800 bags of popcorn?

Price = $\frac{\$880}{800} = \$1.10$

The vendor should charge \$1.10 per bag to break even.

f. When making 800 bags of popcorn, how much should each bag of popcorn be sold at to make a profit of \$720?

Revenue = Costs + Profit

Revenue = $880 + 720 = \$1600$

Price per bag = $\frac{\$1600}{800 \text{ bags}} = \$2/\text{bag}$

The price per bag would be \$2

Or

Cost = \$1.10/bag
Profit = \$0.90/bag

Price = \$2.00/bag

g. How many bags of popcorn can be made with \$6000?

$$6000 = 0.6n + 400$$

$$5600 = 0.6n$$

$$n = \frac{5600}{0.6}$$

$$n = 9333.\overline{3} \approx 9333$$

∴ 9333 bags can be made

Homework:
5.4 # 2-12 even, 15, 16

5.5 Function Notation $y = f(x)$

Recall function notation:

$f(x)$ - f of x

$f(x)$ is used to substitute y

$y = 3x - 4$ linear equation
$f(x) = 3x - 4$ linear function

If $f(x) = 3x - 4$, what does $f(2)$ represent?

To find the value of $f(2)$, we substitute 2 for x in $3x - 4$.

$f(2) = 3(2) - 4$
$f(2) = 6 - 4$
$f(2) = 2$ also means $(2, 2)$

Ex. For $f(x) = 3x - 4$, what value of x gives $f(x) = 26$?

We substitute 26 for $f(x)$
$26 = 3x - 4$
$3x - 4 = 26$
$3x = 30$
$x = 10$

Ex. Given $f(2) = 5$ and $f(-4) = 7$, determine the slope of $f(x)$.

$(2, 5)$ and $(-4, 7)$

$m = \frac{7-5}{-4-2}$

$= \frac{2}{-6}$

$= -\frac{1}{3}$

Ex. Given $f(x) = 3x - 9$, determine the following:

a. $f(4x)$

$= 3(4x) - 9$

$= 12x - 9$

b. $f(x + h)$

$= 3(x + h) - 9$

$= 3x + 3h - 9$

c. $f(x + 1)$

$= 3(x + 1) - 9$

$= 3x + 3 - 9$

$= 3x - 6$

Ex. Given $f(x) = 3x - 2$

Determine $\frac{f(x+h)-f(x)}{h}, h \neq 0$

$f(x) = 3x - 2$

$f(x + h) = 3(x + h) - 2$

$= \frac{f(x+h)-f(x)}{h}$

$= \frac{3(x+h)-2-(3x-2)}{h}$

$= \frac{3x+3h-2-3x+2}{h}$

$= \frac{3h}{h}$

$= 3$

Ex. Given $f(x) = x^2 - 2x$

Determine $\frac{f(x+h)-f(x)}{h}, h \neq 0$

$f(x) = x^2 - 2x$ $\qquad$ $f(x+h) = (x+h)^2 - 2(x+h)$

$$= \frac{f(x+h)-f(x)}{h}$$

$$= \frac{(x+h)^2-2(x+h)-(x^2-2x)}{h}$$

$$= \frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h}$$

$$= \frac{2xh+h^2-2h}{h}$$

$$= 2x + h - 2$$

Homework

5.5 # 1, 2-6 bcf, 7-12 bc, 13 bcf, 14