

PRE-CALCULUS MATHEMATICS 12

A Summary of Basic Identities and Formulae

Sequence and Series:

$$t_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_n = \frac{n}{2}(a + l)$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_n = \frac{a-r^l}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

Sigma Notation:

$$\sum_j^k f(j) = f(j) + f(j+1) + \cdots + f(k)$$

$$n = k - j + 1$$

Combinations:

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

Trigonometry:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Sum and Difference of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Transformations:

$$y = af(b(x - c)) + d$$

$$(x, y) \rightarrow \left(\frac{1}{b}x + c, ay + d\right)$$

Polynomial Division:

$$f(x) = d(x)q(x) + r(x)$$

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

Compound Interest / Exponential Growth and Decay:

$$A = P \left(1 + \frac{r}{n}\right)^{tn}$$

$$A = Pe^{rt}$$

$$A = A_0 x^{\frac{t}{T}}$$

$$A = A_0 e^{kt}$$

Pythagorean Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities:

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Sum and Difference Identities:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Even-Odd and Co-function Identities:

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

Double-Angle Identities:

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= 2 \cos^2 \theta - 1 \\ &= 1 - 2 \sin^2 \theta\end{aligned}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Power-Reducing Identities:

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2 \theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$