Trigonometric Functions Test

Name: Answer Key / 30

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[2] 1. Determine a co-terminal angle with the given θ.

[1] a. $\theta = -135°$

$\theta = -495°, 225°$

[1] b. $\theta = \frac{11\pi}{6}$

$\theta = -\frac{\pi}{6}, \frac{23\pi}{6}$

[2] 2. Determine the quadrant in which θ lies for the following.

[1] a. $\sec\theta > 0, \cot\theta < 0$

$\cos\theta > 0, \tan\theta < 0$

$\therefore$ QIV

[1] b. $\cos\theta < 0, \csc\theta < 0$

$\cos\theta < 0, \sin\theta < 0$

$\therefore$ QIII

[2] 3. Determine the coordinates of the point at the given distance from the origin in the stated quadrant, if θ is its position angle.

[1] a. 10, quadrant IV, $\cos\theta = \frac{1}{5}$

$r = 10 \qquad \cos\theta = \frac{x}{r}$

$\therefore \frac{x}{10} = \frac{1}{5}$

$x = 2$

$y = \pm\sqrt{10^2 - 2^2} = \pm 4\sqrt{6}$

Since θ is in QIV, $y = -4\sqrt{6}$

$(2, -4\sqrt{6})$

[1] b. 4, quadrant III, $\sin\theta = -\frac{1}{2}$

$r = 4 \qquad \sin\theta = \frac{y}{r}$

$\therefore \frac{y}{4} = -\frac{1}{2}$

$y = -2$

$x = \pm\sqrt{4^2 - (-2)^2} = \pm 2\sqrt{3}$

Since θ is in QIII, $x = -2\sqrt{3}$

$(-2\sqrt{3}, -2)$

[2] 4. Find the angle in degrees if an arc length of 5 cm has a radius of 6 cm. Round to nearest degree.

$S = r\theta$

$6\theta = 5$

$\theta = \frac{5}{6}$ rad

$\theta = \frac{5}{6} \times \frac{180°}{\pi}$

$\theta = 48°$

[4] 5. Find all θ, $0 \le \theta < 2\pi$ for the following questions:

[2] a. $\cos\theta + \sqrt{2} = -\cos\theta$

$2\cos\theta = -\sqrt{2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$

$\theta_r = 45°$ or $\frac{\pi}{4}$

$\cos\theta < 0$, θ is in QII and QIII

$\theta_1 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\theta_2 = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

[2] b. $\sin\theta = -0.591$. Round to 3 decimal places.

$\theta = \sin^{-1}(-0.591)$

$\theta = -0.6323$, this angle is in QIV

$\theta_r = 0.6323$

$\sin\theta < 0$, θ is in QIII and QIV

$\theta_1 = \pi + 0.6323 = 3.774$

$\theta_2 = 2\pi - 0.6323 = 5.651$

$\theta = 3.774, 5.651$

[2] 6. Determine the exact value of the following trigonometric functions.

[1] a. $\sin\frac{3\pi}{4}$

θ is in QII, sine ratio is positive

$\theta_r = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

[1] b. $\tan\frac{7\pi}{6}$

θ is in QIII, tangent ratio is positive

$\theta_r = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$

$\tan\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

[4] 7. If θ is an angle in standard position whose terminal side is the graph $6x + 8y = 0, x \ge 0$, determine:

[2] a. Find the values of $\sin\theta$ and $\cos\theta$. Exact answers only.

$8y = -6x$

$y = -\frac{3}{4}x$

this line exists in QII and QIV

since $x \ge 0$, θ is in QIV

a point on $y = -\frac{3}{4}x$ and in QIV: $(4, -3)$

$r = \sqrt{(4)^2 + (-3)^2} = 5$

$\sin\theta = -\frac{3}{5}$ $\cos\theta = \frac{4}{5}$

[2] b. Determine θ to the nearest degree and in radians to 3 decimal places.

$\theta = \sin^{-1}\left(-\frac{3}{5}\right) = 5.640$ radians

$\theta = 5.640 \times \frac{180°}{\pi} = 323°$

[6] 8. For a sine function where a minimum value of -1 occurs at $x = 1$ and the next maximum value of 5 occurs at $x = 3$.

[1] a. Determine the period.

From min to max: $x = 1$ to $x = 3$

$\therefore \frac{1}{2}$ period $= 2$, so period $= 4$

[1] b. Determine the amplitude.

$\text{amp} = \frac{|max - min|}{2} = \frac{|5-(-1)|}{2} = 3$

[2] c. Write an equation in the form $y = a \sin b(x - c) + d$ for the least non-negative real number c, with $a > 0$ and $b > 0$ for the function described above.

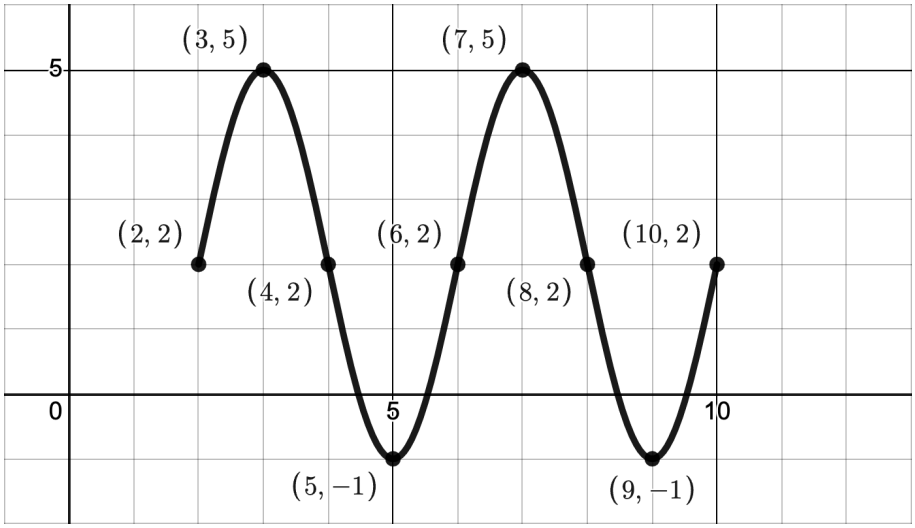
$d = \frac{max+min}{2} = \frac{5+(-1)}{2} = 2$ $\qquad$ $b = \frac{2\pi}{p} = \frac{2\pi}{4} = \frac{\pi}{2}$

For a sine function, the min is ¼ period away from the "beginning" of the next period

$c = 1 + \frac{1}{4}(4) = 2$

$y = 3 \sin \frac{\pi}{2}(x - 2) + 2$

[2] d. Graph 2 periods of the function that is described above. Include all appropriate labels.



[6] 9. A Ferris wheel of radius 28 metres, placed four metres above the ground, varies sinusoidally with time. The Ferris wheel makes one rotation every 30 seconds, with a person sitting four metres from the ground and rising when it starts to rotate.

[2] a. Write a sinusoidal function that describes the function from a person's starting position.

$a = 28 \qquad b = \frac{2\pi}{30} = \frac{\pi}{15} \qquad c = 0 \qquad d = 4 + 28 = 32$

$h(t) = -28\cos\frac{\pi}{15}t + 32$

[1] b. Determine the height above the ground a person would be after 12 seconds, to 1 decimal.

$h(12) = -28\cos\frac{\pi}{15}(12) + 32$
$h(12) = 54.7$ m

[1] c. At what time other time within the first cycle will the Ferris wheel reach the height in part b. Answer to the nearest second.

Since the motion is symmetrical, the Ferris will reach the same height 12 seconds from the end of the period
$t = 30 - 12 = 18$s
or

$54.7 = -28\cos\frac{\pi}{15}t + 32$ $\qquad \frac{\pi}{15}t = \cos^{-1}(-0.809) = 2.513, 3.769$
$22.7 = -28\cos\frac{\pi}{15}t$ $\qquad t = 12, 18$
$\cos\frac{\pi}{15}t = -0.809$ $\qquad \therefore t = 18$s

[2] d. Determine the amount of time the person is more than 50 metres in the air. Answer to the nearest second.

$50 = -28\cos\frac{\pi}{15}t + 32$
$18 = -28\cos\frac{\pi}{15}t$
$\cos\frac{\pi}{15}t = -\frac{9}{14}$
Since cosine ratio is negative, $\frac{\pi}{15}t$ is in QII, QIII
$\frac{\pi}{15}t = \cos^{-1}\left(-\frac{9}{14}\right) = 2.269$ $\qquad \therefore$ reference angle is $\pi - 2.269 = 0.8726$
$\frac{\pi}{15}t = \cos^{-1}\left(-\frac{9}{14}\right) = 2.269, 4.014$ $\qquad$ Angle in QIII $= \pi + 0.8726 = 4.014$
$t = 10.83, 19.17$
Time above 50m $= 19.17 - 10.83 = 8.3\text{s} \approx 8\text{s}$