Chapter 2 – Polynomials

2.1 Classifying Polynomials

Term

A **term** is composed of a coefficient and variable(s) with exponents.

Ex. x $5xy^2$ πr^2 10

x Coefficient is 1, variable is x

$5xy^2$ Coefficient is 5, variables are x and y

πr^2 Coefficient is π, variable is r

10 Coefficient is 10, no variable

Note: terms with no variables are called **constant** terms

Polynomials

A **polynomial** is a mathematical expression involving one or more terms.

In a polynomial, each term must satisfy the following:

- all variables must have whole number exponents
- Terms are separated by "$+$" or "$-$" sign within a polynomial expression

Examples of Polynomials Terms

$9x^2$ $5x^3 + 6y^2$ $x^2 + 2x - 3$

Examples of non-Polynomial Terms:

$x^{-2} = \frac{1}{x^2}$ $x^{\frac{1}{2}} = \sqrt{x}$ $\sqrt{2x} = 2^{\frac{1}{2}}x^{\frac{1}{2}}$

Note: the coefficient must be a real number

Examples of Polynomials with Radical Coefficients

A term can still be polynomial even if the coefficient is a radical

$\sqrt{2}x = 2^{\frac{1}{2}}x$ $\sqrt{35}g^4$ $\sqrt{18}$

$\sqrt{-2}x$ is NOT a polynomial, because $\sqrt{-2}$ is not a real number

Special Names for Polynomials

Monomial: 1 term polynomial

Ex. x $7x^4y$ $-11x^2y^3z$

Binomial: 2 term polynomials

Ex. $x + 1$ $-x + 2y$ $3x^3y - 5xy$

Trinomial: 3 term polynomials

Ex. $a + 2b + 3$ $5x^2 + 2xy + 3y^2$ $5\alpha - 6\beta + 7\delta$

Polynomial: 4 or more terms

Ex. $x + y + z + 1$ $a + 2b + 3c + 4d + 5e + 6$

Degree of a Polynomial

The largest exponent of a term within a polynomial is the **degree of the polynomial**

Ex. Determine the degree of the polynomial $x^3 + 2x^2 + 3x + 4$.

First, determine the degree of each term:

x^3 has a degree of 3 $2x^2$ has a degree of 2

$3x$ has a degree of 1 4 has a degree of 0

Note: constant terms always have a degree of 0

Since the largest degree is **3**, the degree of the polynomial is **3**.

Ex. Determine the degree of the polynomial $x^4 - 5x^2y^3 + 7xy$.

x^4 has a degree of 4

$5x^2y^3$ has a degree of 5

$7xy$ has a degree of 2

∴ the degree of the polynomial is 5

Leading Term and Leading Coefficient

The **leading term** of a polynomial is the term with the largest degree
In the polynomial $x^3 + 2x^2 + 3x + 4$, x^3 would be the leading term.
Since 1 is the coefficient of x^3, the **leading coefficient** is 1.

Collecting Like Terms

Add or subtract terms together that have the exact same variables and exponents

Ex. Simplify the following:

a. $2x - 5x + 6x$

$= 3x$

b. $4x + 5y - 6x + 2$

$= 4x + 5y - 6x + 2$

$= -2x + 5y + 2$

c. $5x^2 + 6x - 3x^2 + 11x$

$= 5x^2 + 6x - 3x^2 + 11x$

$= 2x^2 + 17x$

Evaluating Polynomials

Substituting known values into an expression, and then calculate the value.

Ex. For $x = 2$ and $y = 3$, evaluate $3x^2 + 5xy$.

$$= 3(2)^2 + 5(2)(3)$$

$$= 12 + 30$$

$$= 42$$

Ex. For $x = -3$ and $y = 1$, evaluate $7x^2y + 8xy$.

$$= 7(-3)^2(1) + 8(-3)(1)$$

$$= 63 - 24$$

$$= 39$$

Multiplying Monomials

When multiplying monomials together, first multiply coefficients together, then the variables.

Ex. Simplify $5x(6x^2)$

$$= 30x^3$$

Ex. Simplify $(2x^2y^3z)(-8x^4z^3)$

$$= -16x^6y^3z^4$$

Ex. Simplify $-(-2abc)(-3bc)(-4c)$

$$= 24ab^2c^3$$

Distributive Property $\boldsymbol{a(x+y+z)}$

When a monomial is multiplied to a polynomial, each term of the polynomial is multiplied by the monomial.

Ex. Simplify $a(x+y+z)$

$$= ax + ay + az$$

Ex. Simplify $4(5x+6y)$

$$= 4 \cdot 5x + 4 \cdot 6y$$

$$= 20x + 24y$$

Ex. Simplify $2x^3(5x^2y + 8xy - 4)$

$$= 2x^3 \cdot 5x^2y + 2x^3 \cdot 8xy - 2x^3 \cdot 4$$

$$= 10x^5y + 16x^4y - 8x^3$$

Ex. Simplify $-3x^2(2x^2 + 4x - 8)$

$$= (-3x^2) \cdot 2x^2 + (-3x^2) \cdot 4x - (-3x^2) \cdot 8$$

$$= -6x^4 - 12x^3 + 24x^2$$

2.1 Homework:

3 – 8 bcf…, 9, 10

2.2 Multiplying Polynomials

Distributive Property

This is applied when a polynomial is multiplied to another polynomial

Binomial x Binomial → 2 terms x 2 terms = 4 terms

Ex. Simplify $(x+2)(3x-1)$

First multiply each term in the first set of parentheses x and 2, to each term in the second set of parentheses.

$$= x(3x-1) + 2(3x-1)$$

$$= 3x^2 - x + 6x - 2$$

Then collect like terms

$$= 3x^2 + 5x - 2$$

Binomial x Trinomial → 2 terms x 3 terms = 6 terms

Ex. Simplify $(2x+1)(x^2-x+3)$

$$= 2x(x^2-x+3) + 1(x^2-x+3)$$

$$= 2x^3 - 2x^2 + 6x + x^2 - x + 3$$

$$= 2x^3 - x^2 + 5x + 3$$

Trinomial x Trinomial → 3 terms x 3 terms = 9 terms

Ex. Simplify $(2x+3y+1)(4x-y+2)$

$$= 2x(4x-y+2) + 3y(4x-y+2) + 1(4x-y+2)$$

$$= 8x^2 - 2xy + 4x + 12xy - 3y^2 + 6y + 4x - y + 2$$

$$= 8x^2 + 10xy + 8x - 3y^2 + 5y + 2$$

Binomial Square Formula

$(a+b)^2 = a^2 + 2ab + b^2$ <- both are considered

$(a-b)^2 = a^2 - 2ab + b^2$ <- perfect square trinomials

Ex. Simplify $(2x+3)^2$

Using Distributive Property	Using formula
$= (2x+3)(2x+3)$	Let $a = 2x$ and $b = 3$
$= 2x(2x+3) + 3(2x+3)$	$= (2x)^2 + 2(2x)(3) + (3)^2$
$= 4x^2 + 6x + 6x + 9$	$= 4x^2 + 12x + 9$
$= 4x^2 + 12x + 9$	

Common Mistakes

$(2x+3)^2 = 4x^2 + 9$ this is incorrect‼

$(1+2)^2 = 1 + 4 = 5$ this is incorrect‼

Product of Conjugates

$a+b$ and $a-b$ are called conjugate pairs.
The product of conjugate pairs becomes a difference of squares.

Ex. Simplify $(a+b)(a-b)$

$= a(a-b) + b(a-b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$ <- difference of squares

$\therefore (a+b)(a-b) = a^2 - b^2$

Ex. Simplify $(5x-3)(5x+3)$

Using distributive property

$$= 5x(5x+3) - 3(5x+3)$$

$$= 25x^2 + 15x - 15x - 9$$

$$= 25x^2 - 9$$

Using product of conjugate pairs

$$= (5x)^2 - (3)^2$$

$$= 25x^2 - 9$$

Ex. Simplify $(3a+2)(3a-2)$

$$= (3a)^2 - (2)^2$$

$$= 9a^2 - 4$$

Homework

2.2 # 1 – 4 bcf…, 5, 6-7 bcf

2.3 Removing Common Factors

In any factoring question, always look for GCF first!
Factor the following.

a. $8x - 64$

> re-write as a product of two or more terms

What is the GCF of $8x$ and 64?

8x	64	|	2
4x	32	|	2
2x	16	|	2
x	8		

∴ GCF = 2 x 2 x 2 = 8

$= 8(\text{something})$

something = $\frac{8x}{8} - \frac{64}{8} \div 8 = x - 8$

$= 8(x - 8)$

b. $3x^2 - 12x$

GCF = $3x$

$\frac{3x^2}{3x} - \frac{12x}{3x} = x - 4$

$= 3x(x - 4)$

c. $12x^3y^2 - 16x^2y^2 + 20xy^2$

GCF $= 4xy^2$

$= 4xy^2(3x^2 - 4x + 5)$

d. $3x(x+1)+2(x+1)$

Without substitution

GCF = $x+1$ $\frac{3x(x+1)}{x+1}+\frac{2(x+1)}{x+1}=3x+2$

$=(x+1)(3x+2)$

With substitution

$3x(x+1)+2(x+1)$

let $u=x+1$

$=3xu+2u$

GCF = u

$=u(3x+2)$

$=(x+1)(3x+2)$

e. $4x(x+2)-3(2x+4)$

$=4x(x+2)-3(2)(x+2)$

$=4x(x+2)-6(x+2)$

$=(x+2)(4x-6)$

this is not completely factored, $4x-6$ has a common factor of 2
$=2(x+2)(2x-3)$

Factor by Grouping

a. $x^3 + 4x^2 + 4x + 16$

If there are an even number of terms, you can try to factor by grouping
Break up the polynomial into: $x^3 + 4x^2$ and $4x + 16$

GCF of $x^3 + 4x^2 = x^2$ GCF of $4x + 16 = +4$
$= x^2(x + 4) + 4(x + 4)$

GCF = $x + 4$
$= (x + 4)(x^2 + 4)$

b. $a^2 - 5a + ab - 5b$

$= a(a - 5) + b(a - 5)$

$= (a - 5)(a + b)$

c. $x(x - 1) + 3(1 - x)$

$= x(x - 1) + 3(-x + 1)$

If the first term is a negative, factor out -1
$= x(x - 1) - 3(x - 1)$

$= (x - 1)(x - 3)$

d. $2x^3 - 6x^2 - 9x + 27$

$= 2x^2(x - 3) - 9(x - 3)$

$= (x - 3)(2x^2 - 9)$

e. $x^2 + 2x - 2y - xy$

$= x(x + 2) - y(2 + x)$

Note: $x + 2$ is the same as $2 + x$, but $x - 2$ is not the same as $2 - x$

$= (x + 2)(x - y)$

Or the polynomial could be re-arranged into the correct order

$x^2 + 2x - 2y - xy$

$= x^2 - xy + 2x - 2y$

$= x(x - y) + 2(x - y)$

$= (x - y)(x + 2)$

f. $4x^3 + 8x^2 + 16x + 32$

$= 4(x^3 + 2x^2 + 4x + 8)$

$= 4[x^2(x + 2) + 4(x + 2)]$

$= 4(x + 2)(x^2 + 4)$

Homework

2.3 # 1-6 bcf…, 7, 9, 10, 12

2.4 Factoring $x^2 + bx + c$

Factoring Trinomials Where the Leading Coefficient is Equal to 1

1) Look to factor out the GCF

2) Factor the trinomial: (2 methods)
   - **Decomposition Method** (and Factor by Grouping)

   - **Cross method**

Decomposition Method

To factor $x^2 + bx + c$, need to find:

- Two numbers that add up to coefficient of middle term, b

- Same two numbers must multiply to equal to the product of the coefficients of first and last term, ac

Ex. Factor $x^2 + 5x + 6$

The goal is to re-write the trinomial as a product of binomials.

$x^2 + 5x + 6 \quad \rightarrow \quad (\qquad)(\qquad)$

Find two numbers that add up to 5, and multiply to equal to 6

___ + ___ = 5

___ x ___ = 6

The two numbers are 2 and 3.

Re-write $x^2 + 5x + 6$ where the middle term $5x$ becomes $2x + 3x$

$= x^2 + 2x + 3x + 6$

Next, factor by grouping

$= x(x + 2) + 3(x + 2)$

$= (x + 2)(x + 3)$

Check: To verify answer, simplify $(x+2)(x+3)$ and it should equal to original trinomial x^2+5x+6.

$(x+2)(x+3)$

$=x^2+3x+2x+6$

$=x^2+5x+6$ This matches the original trinomial

Ex. Factor $x^2+5x-24$

___ + ___ $=5$
___ x ___ $=-24$

The two numbers are 8 and -3.

$=x^2+8x-3x-24$

$=x(x+8)-3(x+8)$

$=(x+8)(x-3)$

Factoring Trinomials with a GCF

First, factor out the GCF and proceed with decomposition method.

Ex. Factor $-2x^2+4x+48$

$=-2(x^2-2x-24)$

$-6+4=-2$
$-6\times 4=-24$

$=-2(x^2-6x+4x-24)$

$=-2[x(x-6)+4(x-6)]$

$=-2(x-6)(x+4)$

In addition to factoring out the GCF, also re-arrange polynomial so that the polynomial is in the correct order.

Ex. Factor $3x^2 + 24 - 18x$

Before factoring the trinomial, put in descending order.

$= 3x^2 - 18x + 24$

Then factor out the GCF.

$= 3(x^2 - 6x + 8)$

$-2 + -4 = -6$
$-2 \times\ -4 = 8$

$= 3[x^2 - 2x - 4x + 8]$

$= 3[x(x - 2) - 4(x - 2)]$

$= 3(x - 2)(x - 4)$

Make sure to factor out the negative if the leading term is negative

Ex. Factor $-2x^3 + 18x^2y - 40xy^2$

$= -2x(x^2 - 9xy + 20y^2)$

$-4 + -5 = 5$
$-4 \times -5 = 20$

$= -2x[x^2 - 4xy - 5xy + 20y^2]$

$= -2x[x(x - 4y) - 5y(x - 4y)]$

$= -2x(x - 4y)(x - 5y)$

Using Substitution to Aid Factoring

Ex. Factor $(3a-2b)^2-(6ac-4bc)-24c^2$

First, factor out the GCF from the middle term $6ac-4bc$.

$=(3a-2b)^2-2c(3a-2b)-24c^2$

Notice $3a-2b$ occurs multiple times; use substitution to make the polynomial appear less complex. Write a "Let" statement.

Let $u=3a-2b$

$=u^2-2cu-24c^2$

$-6+4=-2$
$-6\times 4=-24$

$=u^2-6uc+4uc-24c^2$

$=u(u-6c)+4c(u-6c)$

$=(u-6c)(u+4c)$

Replace u with $3a-2b$

$=(3a-2b-6c)(3a-2b+4c)$

Cross Method

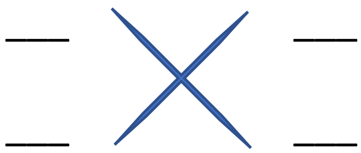
Similar to the decomposition method, cross method factors trinomials in a slightly different way.

The beginning is similar to decomposition where you need to find the two numbers that satisfy the previous criteria.

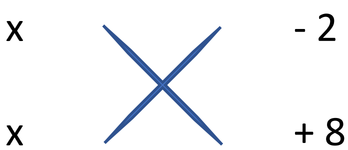
Then, we make a cross where the diagonal products need to equal to the two numbers found previously.

Ex. Factor $x^2 + 6x - 16$

$-2 + 8 = 6$
$-2 \times 8 = -16$



The left blanks must multiply to equal first term, x^2.
The right blanks must multiply to equal last term, -16.



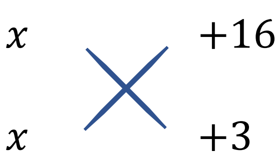
The sum of diagonal products must equal to middle term, $-2x$

The top two represent one set of parentheses, while the bottom two represents the second set.

$= (x - 2)(x + 8)$

Ex. Factor $x^2 + 19x + 48$

$16 + 3 = 19$
$16 \times 3 = 48$



$= (x + 16)(x + 3)$

Short Cut?

For each factoring question, have you noticed link between the "two numbers" and the final answer?

This only applies to $x^2 + bx + c$ trinomials!

Cross Method and Substitution

Ex. Factor $(x + y + 1)^2 + 8(x + y + 1) + 12$

Let $u = x + y + 1$

$= u^2 + 8u + 12$

$6 + 2 = 8$
$6 \times 2 = 12$

x $+6$

x $+2$

$= (u + 6)(u + 2)$

Replace u with $x + y + 1$

$= (x + y + 1 + 6)(x + y + 1 + 2)$

$= (x + y + 7)(x + y + 3)$

2.4 Homework

3-4 ad, 5, 6–11 bcf…, 13, 15, 17, 18

2.5 Factoring $ax^2 + bx + c$

Recall from last lesson:

Decomposition Method

To factor $ax^2 + bx + c$, need to find:

- Two numbers that add up to coefficient of middle term, b

- Same two numbers must multiply to equal to the product of the coefficients of **first** and **last** term, ac

Ex. Factor $2x^2 + 5x + 2$

__ + __ = 5 → 4 + 1 = 5

__ x __ = 4 → 4 x 1 = 4

Decomposition

$= 2x^2 + 4x + 1x + 2$

$= 2x(x + 2) + 1(x + 2)$

$= (x + 2)(2x + 1)$

Cross

$2x$ X $+1$

x $+2$

$= (2x + 1)(x + 2)$

Slide & Divide

The method starts by converting the trinomial into one with a leading coefficient of 1. And then factor as usual using decomposition/cross and then "undoes" the original conversion.

$ax^2 + bx + c \rightarrow \quad x^2 + bx + D$

$2x^2 + 5x + 2 \quad \rightarrow \quad ?$

You multiply (slide) the leading coefficient (2) to the last term.

$2x^2 + 5x + 2 \quad \rightarrow \quad x^2 + 5x + 4$

$x^2 + 5x + 4$

$= (x + 1)(x + 4)$

Now, **divide** the second coefficient in each set of parentheses by the original leading coefficient (2)

$\left(x+\frac{1}{2}\right)\left(x+\frac{4}{2}\right)$ reduce the fraction if possible

$\left(x+\frac{1}{2}\right)\left(x+\frac{2}{1}\right)$ move denominator up front to the first term

$(2x+1)(x+2)$

Ex. Factor $6x^2-13x+6$

__ + __ = -13 → -4 + -9 = -13
__ x __ = 36 → -4 x -9 = 36

2x   -3

3x   -2

$=(2x-3)(3x-2)$

Ex. Factor $-100x^2+120xy-32y^2$

$=-4(25x^2-30xy+8y^2)$

__ + __ = -30 → $-10+-20=-30$

__ x __ = 200 → $-10\times-20=200$

5x   -2y

5x   -4y

$=-4(5x-2y)(5x-4y)$

Ex. Factor $25x^2(a-1)^3 - 5x(a-1)^3 - 2(a-1)^3$

$= (a-1)^3(25x^2 - 5x - 2)$

__ + __ = -5 → 5 + -10 = -5

__ x __ = -50 → 5 x -10 = -50

$= (a-1)^3(25x^2 + 5x - 10x - 2)$

$= (a-1)^3\big(5x(5x+1) - 2(5x+1)\big)$

$= (a-1)^3(5x+1)(5x-2)$

Ex. Factor $4x^{2m} - 20x^m y^n + 25y^{2n}$

→ Same as factoring $4x^2 - 20x^1y^1 + 25y^2$

-10 + -10 = -20
-10 x -10 = 100

$= 4x^{2m} - 10x^m y^n - 10x^m y^n + 25y^{2n}$

$= 2x^m(2x^m - 5y^n) - 5y^n(2x^m - 5y^n)$

$= (2x^m - 5y^n)(2x^m - 5y^n)$

$= (2x^m - 5y^n)^2$

Ex. Factor $3x^{n+2} + 4x^{n+1} - 4x^n$

Recall: $b^{m+n} = b^m \cdot b^n$

So, $x^{n+2} = x^n \cdot x^2$ and $x^{n+1} = x^n \cdot x^1$

$= 3x^n \cdot x^2 + 4x^n \cdot x^1 - 4x^n$

$= x^n(3x^2 + 4x - 4)$

_ + _ = 4
_ x _ = -12

The numbers are -2 and 6

$3x$ -2
$1x$ $+2$

$= x^n(3x - 2)(x + 2)$

2.5 homework
2 – 6 bcf…, 7, 8

2.6 Special Factors

Differences of Squares $a^2 - b^2$

→ can be factored as a product of conjugate pairs

$$a^2 - b^2 = (a + b)(a - b)$$

Factoring a Difference of Squares

Ex. Factor $x^2 - 9$

Since x^2 and 9 are perfect squares, $x^2 - 9$ can be factored as a product of conjugate pairs

$$= (x)^2 - (3)^2$$

$$= (x + 3)(x - 3)$$

Ex. Factor $4x^2 - 81$

$$= (2x + 9)(2x - 9)$$

Ex. Factor $18x^2 - 8y^2$

$$= 2(9x^2 - 4y^2)$$

$$= 2(3x + 2y)(3x - 2y)$$

Ex. Factor $-50ab^2 + 72ac^2$

$$= -2a(25b^2 - 36c^2)$$

$$= -2a(5b + 6c)(5b - 6c)$$

Factoring a Sum of Squares

Ex. Factor $-3x^2 - 27$

$$= -3(x^2 + 9)$$

This expression cannot be factored any further; $x^2 + 9$ is a **sum of squares**.

Force Factoring as a Difference of Squares

Ex. Factor $x^2 - 8$ as a difference of squares

$$= \left(x + \sqrt{8}\right)\left(x - \sqrt{8}\right)$$

$$= \left(x + 2\sqrt{2}\right)\left(x - 2\sqrt{2}\right)$$

Ex. Factor $x - 4$ as a difference of squares

$$= \left(\sqrt{x} + 2\right)\left(\sqrt{x} - 2\right)$$

Ex. Factor $x - 2$ as a difference of squares

$$= \left(\sqrt{x} + \sqrt{2}\right)\left(\sqrt{x} - \sqrt{2}\right)$$

$$= \left(\sqrt{x} + 2\right)\left(\sqrt[4]{x} + \sqrt[4]{2}\right)\left(\sqrt[4]{x} - \sqrt[4]{2}\right)$$

This can go on indefinitely…

Ex. Factor $x^8 - 256$

$$= (x^4 + 16)(x^4 - 16)$$

$$= (x^4 + 16)(x^2 + 4)(x^2 - 4)$$

$$= (x^4 + 16)(x^2 + 4)(x + 2)(x - 2)$$

Perfect Square Trinomial $a^2 + 2ab + b^2$

→ can be factored as a square of a binomial

$$a^2 + 2ab + b^2 = (a + b)^2$$

To check if the trinomial $ax^2 + bx + c$ is a perfect square trinomial, see if $\left(\frac{b}{2}\right)^2 = ac$

Factoring a Perfect Square Trinomial

Ex. Factor $x^2 + 4x + 4$

Check if $\left(\frac{b}{2}\right)^2 = ac$

$$\left(\frac{4}{2}\right)^2 = (1)(4)$$

$$4 = 4$$

∴ this is a perfect square trinomial

$$= (x + 2)^2$$

Ex. Factor $x^2 - 6x + 9$

Check if $\left(\frac{b}{2}\right)^2 = ac$

$$\left(\frac{-6}{2}\right)^2 = (1)(9)$$

$$9 = 9$$

$$= (x - 3)^2$$

Ex. Factor $x^2 + 6x + 8$

Check if $\left(\frac{b}{2}\right)^2 = ac$

$\left(\frac{6}{2}\right)^2 = (1)(8)$

$9 \neq 8$

Cannot factor as a perfect square trinomial, so use decomposition, cross method or slide and divide

Ex. Factor $4x^2 - 12x + 9$

Check if $\left(\frac{b}{2}\right)^2 = ac$

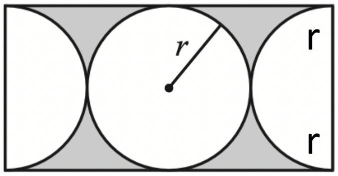
$\left(\frac{-12}{2}\right)^2 = (4)(9)$

$36 = 36$

$= (2x - 3)^2$

2.6 Homework

2, 3-7bcf…, 8, 9 bcf…

8c 

= 2r

= 4r

$A = rectangle - 2circles$
$A = (4r)(2r) - 2\pi r^2$
$A = 8r^2 - 2\pi r^2$
$A = 2r^2(4 - \pi)$

Ex. Factor $(x^6 - 4x^3y^3 + 4y^6) - (a^4 + 6a^2b^2 + 9b^4)$

$(x^3)^2 - 2(x^3)(2y^3) + (2y^3)^2 = (x^3 - 2y^3)^2$

$(a^2)^2 + 2(a^2)(3b^2) + (3b^2)^2 = (a^2 + 3b^2)^2$

$= (x^3 - 2y^3)^2 - (a^2 + 3b^2)^2$

$= (x^3 - 2y^3 + a^2 + 3b^2)\big(x^3 - 2y^3 - (a^2 + 3b^2)\big)$

$= (x^3 - 2y^3 + a^2 + 3b^2)(x^3 - 2y^3 - a^2 - 3b^2)$

$= (a^2 + 3b^2 + x^3 - 2y^3)(-a^2 - 3b^2 + x^3 - 2y^3)$

$= -(a^2 + 3b^2 + x^3 - 2y^3)(a^2 - 3b^2 - x^3 + 2y^3)$

Special Products Extension:

Sum and Difference of Cubes $x^3 + a^3$ and $x^3 - a^3$

$(x^3 + a^3) \div (x + a) = ?$ Guessing a factor for $x^3 + a^3$

$$
\begin{array}{rl}
 & x^2 \quad -ax \quad +a^2 \\
x + a \;| & x^3 + 0x^2 + 0x + a^3 \\
- & (\,x^3 + ax^2\,) \\
 & \quad -ax^2 + 0x \\
 & \quad -(-ax^2 - a^2x) \\
 & \qquad a^2x + a^3 \\
 & \qquad -(\,a^2x + a^3) \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore (x^3 + a^3) = (x + a)(x^2 - ax + a^2)$

Similarly, $(x^3 - a^3) = (x - a)(x^2 + ax + a^2)$

Ex. Factor $x^3 + 1$

$= (x + 1)(x^2 - 1x + 1^2)$

$= (x + 1)(x^2 - x + 1)$

Ex. Factor $x^3 - 1$

$= (x - 1)(x^2 + 1x + 1^2)$

$= (x - 1)(x^2 + x + 1)$

Ex. Factor $x^3 + 8$

$= (x + 2)(x^2 - 2x + 4)$

Ex. Factor $8x^3 - 1$

$= (2x - 1)(4x^2 + 2x + 1)$

2.7 Chapter Review

#2a

Simplify $(2x^2y)(3x^4y^3)$
$= 6x^6y^4$

#5q

Factor $(x+1)^2 - 4(x+1) + 3$
Let $a = x + 1$
$= a^2 - 4a + 3$
$= (a-1)(a-3)$
$= (x+1-1)(x+1-3)$
$= x(x-2)$

#7h

Factor $(x+1)^2 - (y-3)^2$
Let $a = x + 1 \quad b = y - 3$
$= a^2 - b^2$
$= (a+b)(a-b)$
$= (x+1+y-3)\big(x+1-(y-3)\big)$
$= (x+1+y-3)(x+1-y+3)$
$= (x+y-2)(x-y+4)$

#9o

Factor $a^{2n+2} - a^2$
$= a^{2n}a^2 - a^2$
$= a^2(a^{2n} - 1)$
$= a^2(a^n+1)(a^n-1)$

#9g

Factor $x^2(x+10) - 2x(x-8)$
$= x[x(x+10) - 2(x-8)]$
$= x(x^2 + 10x - 2x + 16)$
$= x(x^2 + 8x + 16)$
$= x(x+4)(x+4)$

#9i

Factor $2^{2x} - 2^{x+1} + 1$

$= 2^{2x} - 2^{x}2^{1} + 1$

$= 2^{2x} - 2 \cdot 2^{x} + 1$

Let $a = 2^{x}$

$= a^2 - 2a + 1$

$= (a-1)(a-1)$

$= (a-1)^2$

$= (2^{x} - 1)^2$

2.7 #1-9 bcf…