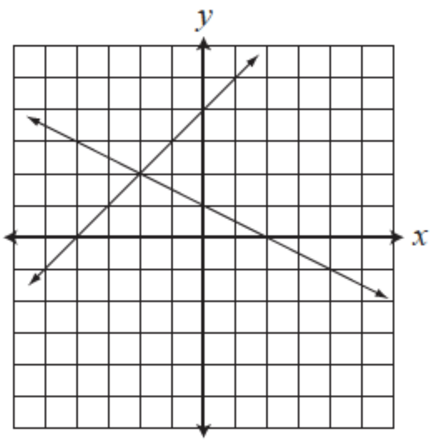
Ch 6 – System of Linear Equations

6.1 Solving Linear Systems by Graphing

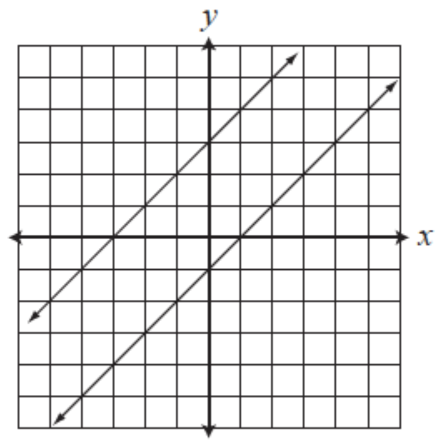
When solving a system of linear equations, there are 3 possible outcomes:

One Solution



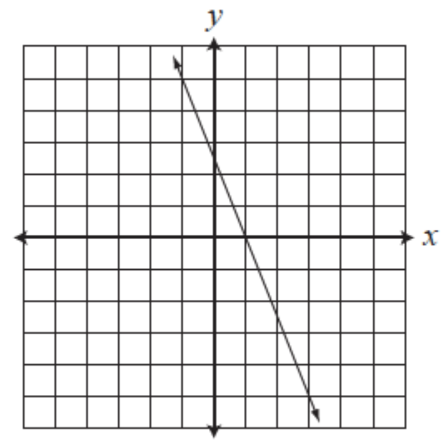
Solution at:
$(-2, 2)$

No Solutions



The lines are ∥
∴ never cross

Infinite Solutions



The lines are
overlapping

Solve by Graphing - The process:

1. Re-write each equation in either slope-intercept form $y = mx + b$ or standard form $Ax + By = C$.

2. if in $y = mx + b$, graph y-int and then use slope to find additional points. If in $Ax + By = C$, determine x-int and y-int, then graph.

3. Identify the point of intersection (the answer)

4. Check answer algebraically

5. Label the point of intersection

Ex. Solve the system by graphing.

(a) $y = -\frac{1}{2}x - 2$

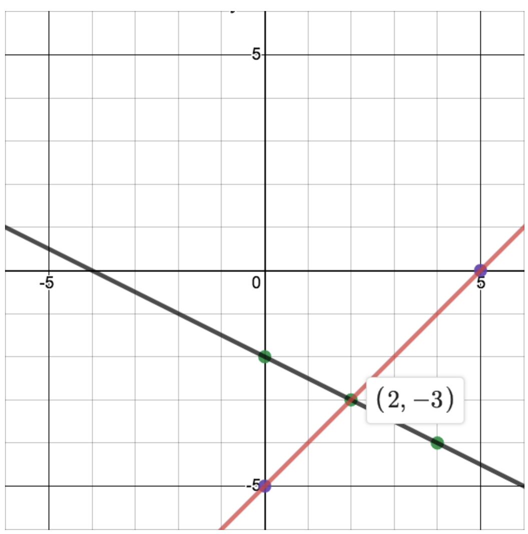
(b) $x - y = 5$

optional convert (b) to slope intercept form

$$y = x - 5$$

For $x - y = 5$ (b)

x-int:	y-int:
$x - 0 = 5$	$0 - y = 5$
$x = 5$	$y = -5$



Check: $(2, -3)$

$$y = -\frac{1}{2}x - 2$$
$$-3 = -\frac{1}{2}(2) - 2$$
$$-3 = -1 - 2$$
$$-3 = -3$$

$$x - y = 5$$
$$2 - (-3) = 5$$
$$5 = 5$$

Since both results in true statements, the solution is correct.

Solving a System of Linear Equations (Parallel Lines)

Parallel lines do not intersect, so there will be no solution.

Ex. Solve by graphing.

$2x - 3y = 3$ (a)

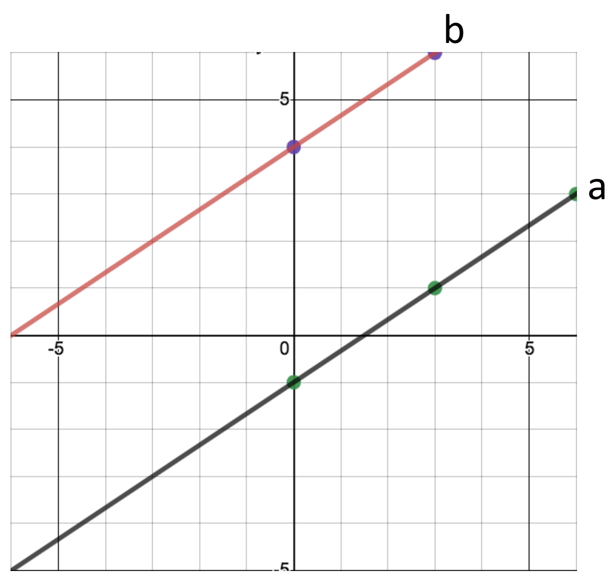
$-2x + 3y = 12$ (b)

Convert each equation to slope intercept form

→ $y = \frac{2}{3}x - 1$ (a)

→ $y = \frac{2}{3}x + 4$ (b)

> the lines have the same slope, but different y-ints, these are parallel lines



The lines a and b are parallel

∴ no solution

Solving a System of Linear Equations (Overlapping Lines)

Overlapping lines are the same line, so there will be an infinite number of solutions.

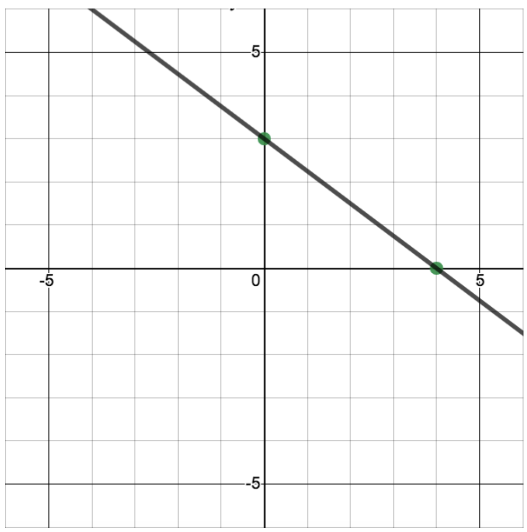
Ex. Solve by graphing

$3x + 4y = 12$ (a)

$y = -\frac{3}{4}x + 3$ (b)

Convert (a) to slope intercept form

$\rightarrow y = -\frac{3}{4}x + 3$ (a)



a, b

> Equations (a) and (b) are identical
>
> ∴ infinite solutions

6.1 Homework

2, 4bcf, 5, 6 bcf…, 7, 9, 11, 14

6.2 Solving Linear Systems by Elimination

When solving a system algebraically, there are two general methods:

1) **Eliminating** a variable
2) Isolating and **Substituting** a variable

Solve By Elimination

In **eliminating** a variable method, we want to make the 2-variable-2-equation system, down to a single variable within a single equation.

→ the preferred form for elimination is standard form, $Ax + By = C$

Ex. Solve the linear system by **elimination**.

$y = -\frac{1}{2}x - 2$ (a)

$x - y = 5$ (b)

Convert (a) to standard form

$$y = -\frac{1}{2}x - 2$$
$$\frac{1}{2}x + y = -2$$
$$x + 2y = -4 \quad \text{(a)}$$

It is preferred to eliminate x for this question, so we will need to subtract the second equation (b) from the first equation (a)

$$\begin{array}{rl} & x + 2y = -4 \\ - & (x - y \quad = 5) \\ \hline & 3y = -9 \quad \text{(c)} \\ & y = -3 \end{array}$$

Now, substitute $y = -3$ into (a) or (b) to solve for x.

$$x - y = 5 \qquad \text{replace } y \text{ with } -3$$
$$x - (-3) = 5$$
$$x + 3 = 5$$
$$x = 2$$

∴ $x = 2, y = -3$ or $(2, -3)$

Ex. Solve $2x - 3y = 2$ (1)

$x + 2y = 8$ (2)

The system needs to be setup so that one of the variables can be eliminated.

> choose to eliminate x, lower common multiple: 2 vs 6

(1) already has a 2 in front of the x; no modifications required

For (2), multiply the whole equation by 2

$2(x + 2y = 8)$

$2x + 4y = 16$ (2)

Re-write the system and subtract the equations in order to eliminate x.

$2x - 3y = 2$ (1)

$-($ $2x + 4y = 16)$ (2)

$-7y = -14$ (3)

$y = 2$

Sub in $y = 2$ into either (1) or (2) to solve for x

$x + 2(2) = 8$ (2)

$x + 4 = 8$

$x = 4$

$\therefore x = 4, y = 2$ or $(4, 2)$

Ex. Solve $\frac{x}{2} - \frac{y}{3} = -\frac{7}{12}$ (1)

$\frac{x}{8} + \frac{y}{9} = 0$ (2)

To remove the fractions, multiply each equation by its LCD.

For (1), the LCD is 12

12 x (1): $6x - 4y = -7$ (1)

For (2), the LCD is 72

72 x (2): $9x + 8y = 0$ (2)

The LCM of x is 18, while LCM of y is 8. Eliminate y.

For (1), multiply by 2.

2 x (1): $12x - 8y = -14$

$12x - 8y = -14$

$+($ $9x + 8y = 0$ $)$

$21x = -14$ (3)

$x = -\frac{14}{21}$

$x = -\frac{2}{3}$

Substitute $x = -\frac{2}{3}$ into $9x + 8y = 0$

$9\left(-\frac{2}{3}\right) + 8y = 0$

$-6 + 8y = 0$

$8y = 6$

$y = \frac{6}{8}$

$y = \frac{3}{4}$

$\therefore x = -\frac{2}{3}, y = \frac{3}{4}$ or $\left(-\frac{2}{3}, \frac{3}{4}\right)$

Exceptions during the solving process

Note: While solving if you get something similar to:
$0 = 5$

Since this is a false statement, the answer is:
No Solution (the two lines in the system are parallel)

If you get something similar to:
$0 = 0$

Since this is a true statement, the answer is:
Infinite Number of Solutions (the two lines are the same)

Solving Systems That Have No Solutions

Ex. Solve $3x - 2y = 1$ (1)
$-6x + 4y = 3$ (2)

2 x (1) $6x - 4y = 2$
(2) $+(-6x + 4y = 3)$

$0 \neq 5$
∴ no solution

Solving Systems That Have Infinite Solutions

Ex. Solve $2x + 5y = 2$ (1)
$-4x - 10y = -4$ (2)

$-\frac{1}{2}\times(2)$ $2x + 5y = 2$
(1) $-(\ 2x + 5y = 2\ )$

$0 = 0$
∴ infinite number of solutions

6.2 Homework:
2 bcf…

Challenge Problem:

Ex. Solve $\frac{3}{x} - \frac{7}{y} = 1$

$\frac{5}{x} + \frac{9}{y} = \frac{7}{5}$

$3\left(\frac{1}{x}\right) - 7\left(\frac{1}{y}\right) = 1$

$5\left(\frac{1}{x}\right) + 9\left(\frac{1}{y}\right) = \frac{7}{5}$

Let $a = \frac{1}{x}$ and $b = \frac{1}{y}$

$3a - 7b = 1$

$5a + 9b = \frac{7}{5}$

6.3 Solving Linear Systems by Substitution

Ex. Solve $6x - y = 0$ (1)

$8x - 3y = 25$ (2)

Isolate the y in (1)

$y = 6x$ (1)

Now, replace the y in (2) with $6x$

$$8x - 3(6x) = 25 \quad (3)$$
$$8x - 18x = 25$$
$$-10x = 25$$
$$\frac{-10x}{-10} = \frac{25}{-10}$$
$$x = -\frac{5}{2}$$

Then substitute $x = -\frac{5}{2}$ into $y = 6x$ to solve for y

$$y = 6\left(-\frac{5}{2}\right)$$
$$y = -15$$

$\therefore$ $x = -\frac{5}{2}$, $y = -15$ or $\left(-\frac{5}{2}, -15\right)$

Ex. Solve $3x - 5y = -14$

$x + 8y = 34$

Ex. Solve $3x + 4y = 11$ (1)
$6x - 5y = -4$ (2)

2 x (1): $6x + 8y = 22$

Now, isolate $6x$

$6x = -8y + 22$ (1)

In (2), replace $6x$ with $-8y + 22$

$(-8y + 22) - 5y = -4$ (3)

$-8y - 5y + 22 = -4$

$-13y = -26$

$\frac{-13y}{-13} = \frac{-26}{-13}$

$y = 2$

and then solve for x…

$6x = -8(2) + 22$

$6x = 6$

$x = 1$

$\therefore x = 1, y = 2$

Alternative substitution solution,

Ex. Solve $3x + 4y = 11$ (1)
$6x - 5y = -4$ (2)

Since $6x$ is a multiple of $3x$:

First isolate $3x$ in (1)

$3x = -4y + 11$

Next, re-write $6x$ in (2), as a product of $3x$ and a number

$2(3x) - 5y = -4$ (2)

Replace the $3x$ with $-4y + 11$

$2(-4y + 11) - 5y = -4$ (3)

$-8y + 22 - 5y = -4$

$-13y = -26$

$y = 2$

Now solve for x

$$3x = -4(2) + 11$$
$$3x = -8 + 11$$
$$3x = 3$$
$$x = 1$$

Ex. Solve by substitution

$\frac{x}{2} - \frac{2y}{3} = 2$ (1)

$\frac{x}{4} + 3y = -4$ (2)

6 x (1) $3x - 4y = 12$

4 x (2) $x + 12y = -16$

Isolate x in (2)

$x = -12y - 16$ (2)

Replace x with $-12y - 16$ in (1)

$3(-12y - 16) - 4y = 12$ (3)

$$-36y - 48 - 4y = 12$$
$$-40y = 60$$
$$y = -\frac{60}{40}$$
$$y = -\frac{3}{2}$$

Sub in $y = -\frac{3}{2}$ into $x = -12y - 16$ and solve for x

$$x = -12\left(-\frac{3}{2}\right) - 16$$
$$x = 18 - 16$$
$$x = 2$$

$\therefore x = 2 \quad y = -\frac{3}{2}$

Ex. If the linear system has one solution, what is the restriction on the value of k.

$$y = 3x + 2$$
$$y = kx + 2$$

To avoid having parallel lines, the slopes cannot be the same.

$$\therefore k \neq 3$$

Ex. Solve the system in terms of a and b for non-zero values of a and b.

$x + ay = b$ (1)

$x - ay = 2b$ (2)

Isolate x from (1)

$$x = b - ay \qquad (1)$$

$$(b - ay) - ay = 2b \qquad (3)$$
$$b - ay - ay = 2b$$
$$-2ay = b$$
$$y = -\frac{b}{2a}$$

$$x = b - a\left(-\frac{b}{2a}\right)$$
$$x = b + \frac{b}{2}$$
$$x = \frac{3b}{2} \qquad \text{or} \qquad x = \frac{3}{2}b$$
$$y = -\frac{b}{2a} \qquad\qquad y = -\frac{b}{2a}$$

6.3 Homework

1 bcf…, 2-4, 5 bcd

Extra Solving Practice

1. $2x - 3y = 15$ (1)

   $5x - 2y = 10$ (2)

$2x = 3y + 15$

$x = \frac{3}{2}y + \frac{15}{2}$ (1)

$5\left(\frac{3}{2}y + \frac{15}{2}\right) - 2y = 10$ (3)

$\frac{15}{2}y + \frac{75}{2} - 2y = 10$

$\frac{11}{2}y = -\frac{55}{2}$

$y = -5$

$x = \frac{3}{2}(-5) + \frac{15}{2}$

$x = -\frac{15}{2} + \frac{15}{2}$

$x = 0$

$x = 0, y = -5$

2. $3x - 2y = 4$ (1)

   $20x + 0.5y = 15$ (2)

(2) x 2 $40x + y = 30$

$y = 30 - 40x$ (2)

$3x - 2(30 - 40x) = 4$ (3)

$3x - 60 + 80x = 4$

$83x = 64$

$x = \frac{64}{83}$

$y = 30 - 40\left(\frac{64}{83}\right)$

$y = 30 - \frac{2560}{83}$

$y = -\frac{70}{83}$

$x = \frac{64}{83}, y = -\frac{70}{83}$

3. $\frac{2(x-1)}{5} + \frac{y+4}{3} = 17$ (1)

$\frac{3(2x+1)}{4} - \frac{5(y-2)}{6} = 31$ (2)

15 x (1) $3(2)(x-1) + 5(y+4) = 255$

$6x - 6 + 5y + 20 = 255$

$6x + 5y = 241$ (1)

12 x (2) $3(3)(2x+1) - 2(5)(y-2) = 372$

$18x + 9 - 10y + 20 = 372$

$18x - 10y = 343$ (2)

2 x (1) $12x + 10y = 482$

\+ $18x - 10y = 343$

$30x = 825$

$x = \frac{55}{2}$

$6\left(\frac{55}{2}\right) + 5y = 241$

$165 + 5y = 241$

$5y = 76$

$y = \frac{76}{5}$

$x = \frac{55}{2}, y = \frac{76}{5}$

MP11 BLM 1.3 #1-13 odd, 14-16

1.5 #1-15 odd, 17-19

6.4 Problem Solving with Two Variables

Ex. Adult tickets from the school play are \$12.00 and children's tickets are \$8.00. If a theatre holds 300 seats and the sold-out performance brings in \$3280.00, how many children and adults attended the play?

Let $a =$ # of adults attending
$c =$ # of children attending

$a + c = 300$ (1)
$12a + 8c = 3280$ (2)

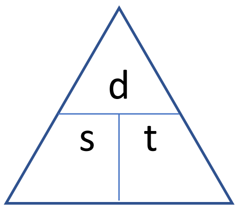
$a = 300 - c$ (1)

$12(300 - c) + 8c = 3280$ (3)
$3600 - 12c + 8c = 3280$
$-4c = -320$
$c = 80$
$a = 300 - 80 = 220$

∴ 220 adults and 80 children attended the school play

Ex. A small airplane makes a 2400 km trip in 7.5 hours, and makes the return in 6 hours. If the plane travels at a constant speed, and the wind blows at a constant rate, find the airplane's airspeed, and the speed of the wind.

Let $p =$ speed of the plane
$w =$ speed of the wind



$d = st$

	d	s	t
With wind	2400	$p + w$	6
Against wind	2400	$p - w$	7.5

With wind:

$$2400 = (p + w)(6)$$
$$2400 = 6p + 6w$$
$$400 = p + w \qquad (1)$$

Against wind:

$$2400 = (p - w)(7.5)$$
$$2400 = 7.5p - 7.5w$$
$$320 = p - w \qquad (2)$$

$$p + w = 400 \qquad (1)$$
$$+ \ (\ p - w = 320\ ) \qquad (2)$$

$$2p = 720 \qquad (3)$$
$$p = 360$$

$$360 + w = 400$$
$$w = 400 - 360$$
$$w = 40$$

∴ the speed of the plane is 360 km/h while the speed of the wind is 40 km/h

Ex. Brand A fertilizer is 32% phosphorus, while Brand B is 18% phosphorus. How much of each is needed to produce 56 t of a 24% mixture?

Ex. An oil tanker can be filled in 18 h using one pump, and in 15 h using a different pump. If both pumps are used, how long would it take to fill the tanker?

Ex. One train leaves a station heading west. A second train, heading east, leaves the same station 2 h later and travels 15 km/h faster than the first. They are 580 km apart 6 h after the second train departed. How fast is each train travelling?

6.4 Homework

1bcf…, 3, 4, 6, 8, 10, 13, 15, 16, 18, 19

6.5 Arithmetic Sequence

Sequence (Patterns)

Examples:	1, 1, 2, 3, 5, 8, 13, 21, …	Fibonacci Sequence
	1, 2, 3, 4, 5, 6, …	Arithmetic sequence
	-5, -6, -7, -8, …	Arithmetic sequence
	1, 2, 4, 8, 16, …	Geometric sequence

Finite Sequence is a pattern that has a beginning and an end.
Infinite Sequence has a beginning, but goes on forever.

The **first term** in a sequence is denoted $\boldsymbol{a}$ or t_1 or a_1
For our class, we will use a and t_1 interchangeably

Ex. Predict the next three terms of the sequence.

a. 1, 5, 9, …

each term skips / jumps by 4
13, 17, 21

b. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

$\frac{4}{5}, \frac{5}{6}, \frac{6}{7}$

c. $5, 8, 13, 21, \ldots$

$34, 55, 89$

Part of the Fibonacci sequence

Arithmetic Sequence

A pattern where the terms are separated by adding (or subtracting) a **common difference**, $\boldsymbol{d}$.

Ex. Determine the common difference in the following arithmetic sequence

a. 5, 1, -3, -7

Each term is separated by -4
∴ the common difference is -4

b. 20, 30, 40, 50,…

Each term is separated by +10
∴ the common difference is 10

c. 2, __, __, 11

3 common differences is equivalent to +9
So, each term is separated by +3
∴ the common difference is 3

The General Term for an Arithmetic Sequence

The general term by

$$t_n = a + (n-1)d$$

where,

t_n the term at position n

a the 1st term (the term at position 1)

n end position n (the number of terms in a sequence)

d the common difference

Ex. Determine the first 3 terms of the sequence

General Term

a. $t_n = 5 + (n - 1)6$

1st term: $n = 1$ $t_1 = 5 + (1 - 1)6$

$t_1 = 5 + 0 = 5$

2nd term: $n = 2$ $t_2 = 5 + (2 - 1)6$

$t_2 = 5 + 6 = 11$

3rd term: $n = 3$ $t_3 = 5 + (3 - 1)6$

$t_3 = 5 + 12 = 17$

5, 11, 17

Simplified General Term

b. $t_n = 3n + 4$

$n = 1$ $t_1 = 3(1) + 4 = 3 + 4 = 7$

$n = 2$ $t_2 = 3(2) + 4 = 6 + 4 = 10$

$n = 3$ $t_3 = 3(3) + 4 = 9 + 4 = 13$

$7, 10, 13$

Ex. Find the 75th term of the sequence $-3, 2, 7, \ldots$

The general term: $t_n = a + (n-1)d$

$n = 75$ $a = -3$ $d = t_2 - t_1 = 5$

In general, $d = t_n - t_{n-1}$ to find common difference, subtract any 2 consecutive terms

$t_n = a + (n-1)d$

$t_n = -3 + (n-1)(5)$

$t_{75} = -3 + (75-1)(5)$

$t_{75} = -3 + 74(5)$

$t_{75} = -3 + 370$

$t_{75} = 367$

Using Simplified t_n

$\rightarrow$ $t_n = 5n - 8$

$t_{75} = 5(75) - 8$

$t_{75} = 375 - 8$

$t_{75} = 367$

Ex. Which term in the arithmetic sequence 10, 6, 2,… has a value of -390?

$a = 10$ $n = ?$ $d = -4$ $t_n = -390$

$-390 = 10 + (n-1)(-4)$

Solve for n

$-390 = 10 - 4(n-1)$

$-400 = -4(n-1)$

$100 = n - 1$

$n = 101$

∴ The 101st term is -390

Ex. The 7th term of arithmetic sequence is 78, and the 18th term is 45. Determine the 3rd term.

$t_7 = 78$ $\quad\quad$ $t_{18} = 45$ $\quad\quad$ $t_3 = ?$

Sub in our term values into our general term equation

$$t_n = a + (n-1)d$$

$t_7 = 78 \rightarrow \quad 78 = a + (7-1)d$

$78 = a + 6d \quad (1)$

$t_{18} = 45 \rightarrow \quad 45 = a + (18-1)d$

$45 = a + 17d \quad (2)$

$78 = a + 6d$

$-\ (45 = a + 17d)$

$33 = -11d$

$d = -3$

$45 = a + 17(-3)$

$45 = a - 51$

$a = 96$

$t_3 = 96 + (3-1)(-3)$

$t_3 = 96 - 6$

$t_3 = 90$

∴ the 3rd term is equal to 90

Ex. Find x so that $3x + 2$, $2x - 3$, and $2 - 4x$ are consecutive terms of an arithmetic sequence.

$d = t_n - t_{n-1}$ common difference is subtraction of 2 consecutive terms

$d = t_2 - t_1$
$d = 2x - 3 - (3x + 2)$
$d = -x - 5$

$d = t_3 - t_2$
$d = 2 - 4x - (2x - 3)$
$d = -6x + 5$

$\therefore \quad -x - 5 = -6x + 5$
$5x = 10$
$x = 2$

To confirm: $3(2) + 2 = 8$
$2(2) - 3 = 1$
$2 - 4(2) = -6$

$8, 1, -6$

Homework

6.5 #7-11 bcf, 12, 15, 16, 19

6.6 Arithmetic Series

Sigma Notation

To write the **sum** of a sequence of terms, we can use the sigma notation.

$$t_1 + t_2 + t_3 + \cdots + t_n = ?$$

Using sigma notation, the sum the terms above can be written as:

$$\sum_{k=1}^{n} t_k$$

n is the position of the last term

t_k is the expression for the general term

k is position of the first term

$n - k + 1$ is the number of terms

Ex. Determine the sum of each sequence

$$\sum_{k=1}^{10} (2k - 9)$$

$$t_1 = 2(1) - 9 = -7$$

$$t_2 = 2(2) - 9 = -5$$

$$\dots$$

$$t_{10} = 2(10) - 9 = 11$$

$$= (-7) + (-5) + \cdots + 11$$

$$= 5 \times 4 = 20$$

Ex. Write in sigma notation for $5 + 7 + 9 + \cdots + 31$

$$\sum_{k=1}^{n} t_k$$

First find an expression for t_k

$t_1 = a = 5 \qquad d = 2$

$t_n = a + (n-1)d$

$t_n = 5 + (n-1)(2)$

$t_n = 5 + 2n - 2$

$t_n = 2n + 3$

Next, find the position of the last term

$31 = 5 + (n-1)(2)$

$26 = 2(n-1)$

$13 = n - 1$

$n = 14$

Finally, write the Sigma Notation for the sum

$$\sum_{k=1}^{14} (2k + 3)$$

Homework:
6.5 # 5, 6

Arithmetic Series

Instead of manually adding up the terms of a series, there is a formula that gives you the result in a simple and concise format.

S_n refers to the sum of all the terms from t_1 to t_n

$$
\begin{array}{rl}
 & S_n = a + t_2 + t_3 + \cdots + t_{n-2} + t_{n-1} + t_n \\
+ & S_n = t_n + t_{n-1} + t_{n-2} + \cdots + t_3 + t_2 + a \\
\hline
 & 2S_n = \underbrace{(a + t_n) + \cdots + (a + t_n)}_{n \text{ times}}
\end{array}
$$

Note: a is the first term, while $l = t_n$ which is the last term

$$2S_n = n(a + l)$$

$$\therefore S_n = \frac{n}{2}(a + l)$$

Since $l = t_n$ and $t_n = a + (n-1)d$, substituting this into the formula above:

$$S_n = \frac{n}{2}(a + a + (n-1)d)$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

In summary, the sum of an arithmetic series can be found using:

$$S_n = \frac{n}{2}(2a + (n-1)d) \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

Ex. For the sum of $5 + 7 + 9 + 11 + 13 + 15 + 17$

a. Find the sum

Method 1: using $S_n = \frac{n}{2}(2a + (n-1)d)$

$n = 7 \qquad d = 2 \qquad a = 5$

$S_7 = \frac{7}{2}\big(2(5) + (7-1)(2)\big)$

$= 77$

Method 2: using $S_n = \frac{n}{2}(a + l)$

$a = 5 \qquad l = 17 \qquad n = 7$

$S_7 = \frac{7}{2}(5 + 17)$

$= 77$

b. Write the sum in sigma notation

Need to find the simplified expression for t_n

$t_n = 5 + (n-1)(2)$

$t_n = 5 + 2n - 2$

$t_n = 2n + 3$

Note: the number of terms $= n - k + 1$

$$\sum_{k=1}^{7}(2k + 3)$$

Ex. Determine the sum of odd integers from 0 to 300.

$$S_n = 1 + 3 + \cdots + 299$$

$$a = 1 \qquad d = 2 \qquad n = ? \qquad l = 299$$

$$299 = 1 + (n-1)(2)$$

$$298 = 2(n-1)$$

$$149 = n - 1$$

$$n = 150$$

$$S_{150} = \frac{150}{2}(1 + 299)$$

$$S_{150} = 22500$$

Ex. Find the following sum

$$\sum_{k=1}^{281}(-3k + 85)$$

$$a = 82 \qquad d = -3 \qquad n = 281$$

$$l = t_{281}$$

$$t_{281} = -3(281) + 85 = -758$$

$$S_{281} = \frac{281}{2}\left(82 + (-758)\right)$$

$$S_{281} = -94978$$

Ex. If the sum of the terms of an arithmetic series is 234, and the middle term is 26, find the number of terms in the series.

$S_n = 234$ $S_n = a + \cdots + 26 + \cdots + t_n$

There are n terms, but without the middle term, there would only be $n - 1$ terms. Considering a and l as one pair, there would $\frac{n-1}{2}$ pairs that share the same sum.

$S_n = \frac{n-1}{2}(a + l) + M$ Let $M =$ middle term

$a + l = 2M$

$\therefore 234 = \frac{n-1}{2}\big(2(26)\big) + 26$

$208 = \frac{n-1}{2}(52)$

$4 = \frac{n-1}{2}$

$8 = n - 1$

$n = 9$ $\therefore$ there are 9 terms in this series

Homework

6.6 # 1-4 bcf…, 7, 9, 12, 13

Challenge Question

Find the sum.

$$\sum_{k=1}^{t}(-3k + x^2)$$

Number of terms $= t - 1 + 1 = t$

$a = -3 + x^2$ $\quad$ $l = -3t + x^2$ $\quad$ $n = t$

$$S_n = \frac{n}{2}(a + l)$$

$$S_t = \frac{t}{2}(-3 + x^2 - 3t + x^2)$$

$$S_t = \frac{t}{2}(-3t + 2x^2 - 3)$$

$$S_t = -\frac{3}{2}t^2 + tx^2 - \frac{3}{2}t$$