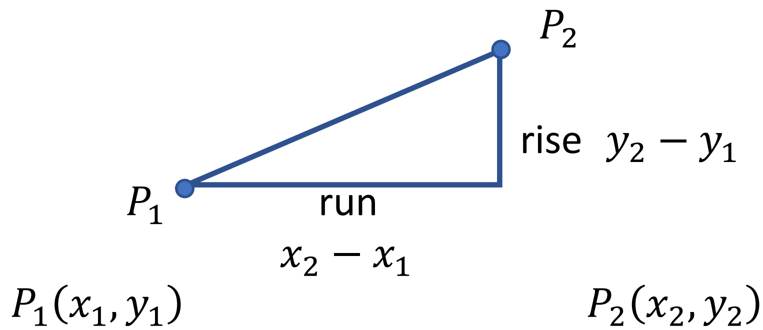
Chapter 4 – Linear Functions

4.1 – Slope

Slope: $m = \frac{rise}{run}$

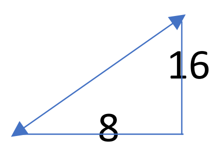


$m = \frac{y_2 - y_1}{x_2 - x_1}$ Equation of the slope for a linear function

Ex. Given two coordinates, determine the slope between the points.

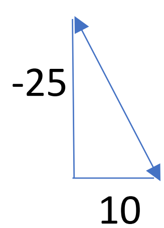
a. $(-2,4)$ and $(6, 20)$

$m = \frac{20-4}{6-(-2)} = \frac{16}{8} = 2$



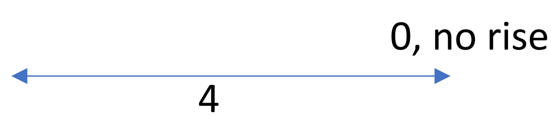
b. $(8, -5)$ and $(-2, 20)$

$m = \frac{20-(-5)}{-2-8} = \frac{25}{-10} = -\frac{5}{2}$



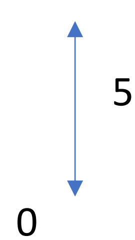
c. $(6,7)$ and $(10,7)$

$m = \frac{7-7}{10-6} = \frac{0}{4} = 0$



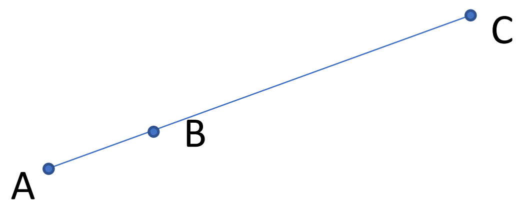
d. $(-11, 3)$ and $(-11, 8)$

$m = \frac{8-3}{-11-(-11)} = \frac{5}{0} =$ undefined



Coordinates on a Line

Points A, B, and C are coordinates on a line; the make line segments AB, AC, and BC. The slopes of each line segment are exactly the same.



$$m_{AB} = m_{AC} = m_{BC}$$

Ex. Points $A(8, 3)$, $B(2, 4)$, and $C(x, -19)$ are coordinates on the same line. Determine x.

First find m_{AB}

$$m_{AB} = \frac{4-3}{2-8} = -\frac{1}{6}$$

Since all three coordinates are on the same line, $m_{AB} = m_{BC}$

$$\therefore m_{BC} = -\frac{1}{6}$$

Find expression for m_{BC}

$$m_{BC} = \frac{-19-4}{x-2} = -\frac{23}{x-2}$$

Equate the value of m_{BC} and the expression

$$-\frac{1}{6} = -\frac{23}{x-2}$$

Cross multiply and solve for x.

$$x - 2 = (6)(23)$$

$$x - 2 = 138$$

$$x = 140$$

Slope and Steepness

The "greater" the slope, the "steeper" it is.

Ex. $m = 2$ is steeper than $m = \frac{1}{2}$

$$2 > \frac{1}{2}$$

$m = -2$ is not steeper than $m = -4$, even though $-2 > -4$.
Why?

use $|m|$ when comparing the steepness of a line

$$|-4| > |-2| \quad \rightarrow \quad 4 > 2$$

$\therefore m = -4$ is steeper than $m = -2$

Ex. Arrange the slopes from flattest to steepest. $-3, \frac{2}{3}, -\frac{5}{3}, 2$

Applying the $|\ \ |$ → $3, \quad \frac{2}{3}, \quad \frac{5}{3}, \quad 2$

$\therefore \frac{2}{3}, -\frac{5}{3}, 2, -3$

4.1 Homework

#1-6 bcf…, 7, 10 – 12

4.2 Rate of Change

Rate is the same as the slope (rate is used in word problems)

Rate of Change The comparison of two dissimilar quantities

Examples

Speed:	Kilometres per hour	km/h
	Metres per second	m/s
Fuel Economy:	Litres per 100 kilometres	L/100km
	Miles Per Gallon	mpg
Wages:	dollars per hour	$/hr

Ex. In the city of Richland, they experienced an increase of population of 24000 people over 6 years. What was the rate of change?

Rate of change: $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$\Delta x =$ change in x value

$\Delta y =$ change in y value

$= \frac{24000 \text{ people}}{6 \text{ years}}$

$= 4000 \text{ people/yr}$ (unit rate)

Ex. Jon runs 500 m in 20 seconds. Determine his speed.

Rate of change $= \frac{500 \text{ m}}{20 \text{ sec}}$

$= 25 \text{ m/s}$ (unit rate)

Ex. Tala is calculating her fuel consumption. Her car's odometer read 24567 km at the beginning of her week. At the end of the week, the odometer read 26123 km. She used 100 L of gas. The cost of fuel is 190 cents per litre.

a. Determine the rate of gas consumption for the car-tank (km/L)

Rate = $\frac{distance}{fuel\ used} = \frac{26123-24567}{100} = \frac{1556}{100}$

$= 15.56 \approx 15.6$ km / L

b. Determine her rate of driving per day (km/day)

Rate = $\frac{distance}{days} = \frac{1556}{7}$

$= 222.3$ km/day

c. Determine Tala's cost per day for fuel.

Rate = $\frac{cost}{days} = \frac{100 \times 1.90}{7} = \$27.14/day$

Ex. Shelly sells seashells by the seashore. She is paid a basic monthly salary of \$1500, plus \$4 for every seashell she sells.

a. Write an expression for her monthly earnings, E, depending on the number of seashells, s, she sells.

$E = ?$

Rate = $\frac{\$4}{seashell}$

Recall: $y = mx + b$

$E = 4s + 1500$

b. Determine how many seashells Shelly would have to sell in order to make $2700 in a month.

$E = 2700 \qquad s = ?$

$2700 = 4s + 1500$

$1200 = 4s$

$s = 300$

∴ Shelly must sell 300 seashells to earn $2700 a month

c. Determine Shelly's earnings if she sells 1200 seashells in a month.

$E = ? \qquad s = 1200$

$E = 4(1200) + 1500 \qquad E(s) = 4s + 1500$ function notation

$E = 4800 + 1500$

$E = 6300$

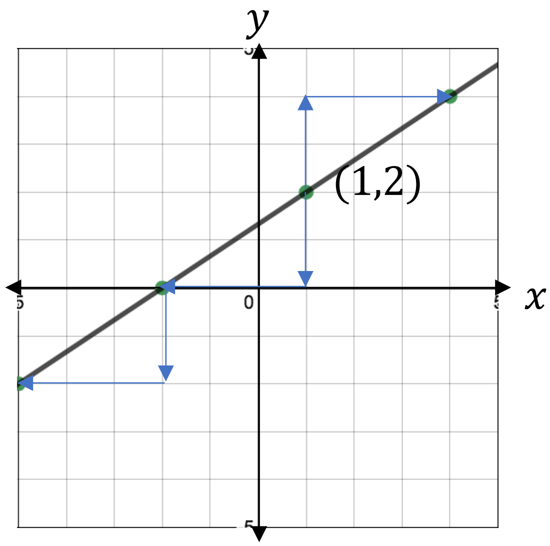
∴ Shelly makes $6300 if she sells 1200 seashells

4.2 Homework

#1-4 bcf…, 6, 8, 10, 11, 14, 15, 17, 19

4.3 – Graphing Linear Functions

Ex. Graph a line with slope of $\frac{2}{3}$, going through the point $(1, 2)$.



First, plot the coordinate $(1, 2)$

Then using the slope: $\frac{2}{3}$ up 2, and right 3 to find additional coordinates

Then using the slope again: $\frac{-2}{-3}$ down 2, left 3 to find additional coordinates

Finally, connect all coordinates with a line using a ruler

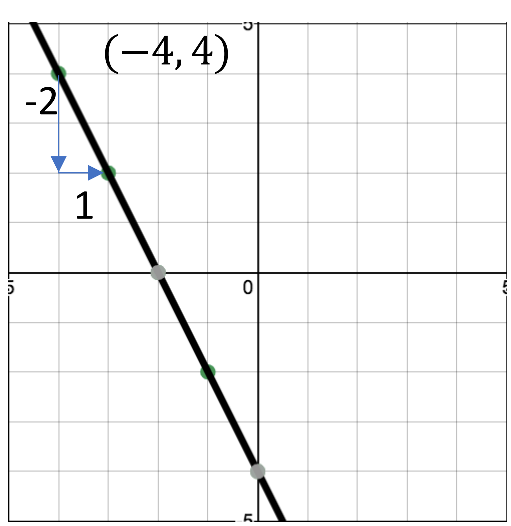
Ex. Graph a line with slope -2, going through the point $(-4, 4)$.

First graph the coordinate $(-4, 4)$

Then using the slope, find additional coordinates:

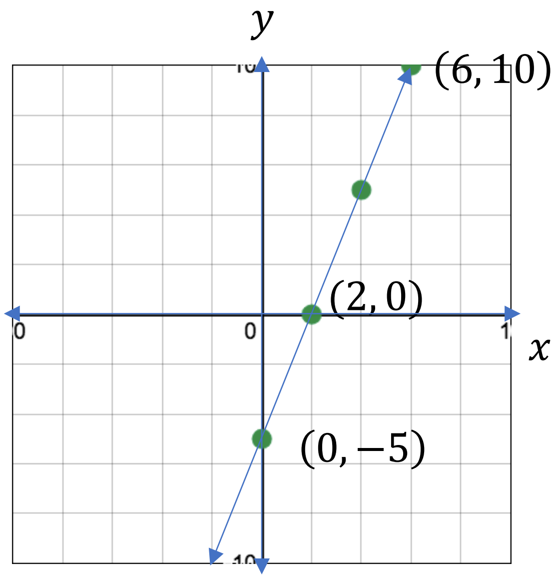
$m = -2 = \frac{-2}{1}$ or $\frac{2}{-1}$

So, down 2 and right 1, or up 2 and left 1



Ex. Determine the x-intercept and y-intercept of a linear equation with the slope $\frac{5}{2}$ and going through $(6, 10)$.

Solve by graphing:



First, plot the coordinate $(6, 10)$

Then, use the slope to find additional points

$m = \frac{5}{2}$ or $\frac{-5}{-2}$

up 5 and right 2 or down 5 and left 2

$\therefore$ x-int is $(2, 0)$ and y-int is $(0, -5)$

Algebraic Solution to the problem above:

First determine the equation of the line in slope-intercept form:

$y = mx + b$ $\qquad$ $m = \frac{5}{2}$

$y = \frac{5}{2}x + b$

Plug in $(6, 10) \rightarrow$ $\qquad$ $x = 6, y = 10$

$10 = \frac{5}{2}(6) + b$

$10 = 15 + b$

$b = -5$ $\quad$ which means that the y-int is $(0, -5)$

$\therefore y = \frac{5}{2}x - 5$

To solve for x-int, sub in $y = 0$ into $y = \frac{5}{2}x - 5$

$0 = \frac{5}{2}x - 5$

$-\frac{5}{2}x = -5$

$-5x = -10$ $\qquad$ multiply by LCD = 2

$\frac{-5x}{-5} = \frac{-10}{-5}$

$x = 2$

$\therefore$ x-int is $(2, 0)$

Ex. A line is parallel to $7x - 12y = 13$ and passes through $(-15, 16)$. Determine the equation of the line in slope-intercept form.

Find parallel slope:

$$-12y = -7x + 13$$

$$\frac{-12}{-12}y = \frac{-7}{-12}x + \frac{13}{-12}$$

$$y = \frac{7}{12}x - \frac{13}{12}$$

$$\therefore m_{\parallel} = \frac{7}{12}$$

Find the linear equation:

$$16 = \frac{7}{12}(-15) + b$$

$$16 = -\frac{35}{4} + b$$

$$b = \frac{99}{4}$$

$$\therefore y = \frac{7}{12}x + \frac{99}{4}$$

Ex. A line is parallel to $7x - 12y = 13$ and passes through $(-15, 16)$. Determine the equation of the line in **standard form**.

In standard form, parallel lines have the same A and B values

∴ Parallel Line: $7x - 12y = C$

Plug in $(-15, 16)$ into $7x - 12y = C$ and solve for C

$$7(-15) - 12(16) = C$$

$$C = -297$$

$$\therefore 7x - 12y = -297$$

Ex. Find the equation of the line that passes through $(5,11)$ and $(13,4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-11}{13-5} = -\frac{7}{8}$$

$$y = mx + b$$

$$11 = -\frac{7}{8}(5) + b$$

$$11 = -\frac{35}{8} + b$$

$$b = \frac{123}{8}$$

$$\therefore y = -\frac{7}{8}x + \frac{123}{8}$$

Ex. Determine the slope of a linear function that has points $(2a, 6b)$ and $\left(8b, \frac{3}{2}a\right)$.

$$m = \frac{\frac{3}{2}a - 6b}{8b - 2a} \times \frac{2}{2}$$

$$m = \frac{3a - 12b}{16b - 4a}$$

$$m = \frac{3(a-4b)}{-4(a-4b)}$$

$$m = -\frac{3}{4}$$

4.3 Homework

1-6 bcf..., 8, 9, 12, 14

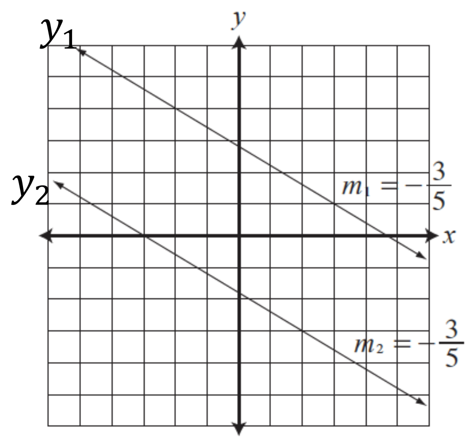
4.4 - Parallel and Perpendicular Lines

Parallel Lines

- lines that never meet
- lines that have the same slope
  ("∥" means parallel)
- have different y intercepts

$m_1 = m_2$

$y_1 \parallel y_2$



Perpendicular Lines

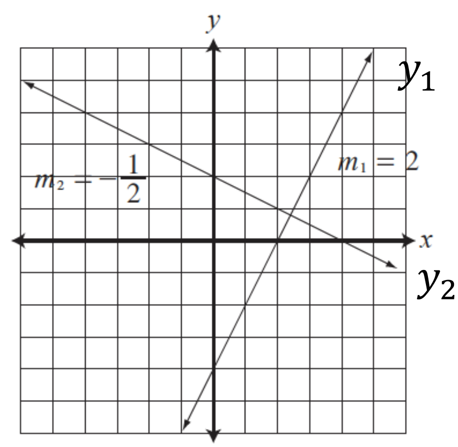
- Lines that meet at a 90° angle
- Lines that have slopes that are negative reciprocals of each other

(⊥ means perpendicular)

if $y_1 \perp y_2$

then $m_1 \times m_2 = -1$

$m_1 = -\frac{1}{m_2}$



Ex. Parallel, Perpendicular, or neither?

a. L_1: $(-2, 7), (6, 1)$ L_2: $(5, -3), (13, -9)$

$m_1 = \frac{1-7}{6-(-2)} = -\frac{6}{8} = -\frac{3}{4}$ $m_2 = \frac{-9-(-3)}{13-5} = -\frac{6}{8} = -\frac{3}{4}$

∴ the lines are parallel because the slopes are the same

b. L_1: $(-x, 3y), (4x, -2y)$ L_2: $(6y, 3x), (-2y, -5x)$

$m_1 = \frac{-2y-3y}{4x-(-x)} = \frac{-5y}{5x} = -\frac{y}{x}$ $m_2 = \frac{-5x-3x}{-2y-6y} = \frac{-8x}{-8y} = \frac{x}{y}$

$-\frac{y}{x} \cdot \frac{x}{y} = -1$

∴ the lines are perpendicular

Ex. The line through $(x, -6)$ and $(-2, -1)$ is perpendicular to a line with slope of -2. Determine x.

The perpendicular slope is -2, so the slope of the line we are working with is $\frac{1}{2}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\qquad$ $m \perp -2$

$m = \frac{-1-(-6)}{-2-x} = \frac{5}{-2-x}$ $\qquad$ $m = -\frac{1}{-2} = \frac{1}{2}$

$\therefore \frac{1}{2} = \frac{5}{-2-x}$

Cross multiply and solve for x.

$-2 - x = 10$

$-x = 12$

$x = -12$

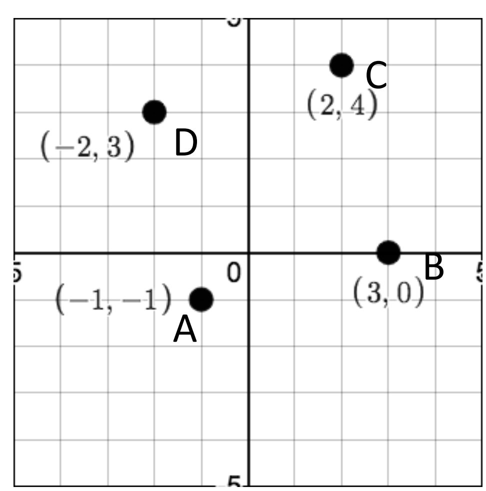
Ex. Show that the points $A(-1, -1), B(3, 0), C(2, 4)$ and $D(-2, 3)$ are the vertices of a rectangle.

$m_{AB} = \frac{0-(-1)}{3-(-1)} = \frac{1}{4}$

$m_{BC} = \frac{4-0}{2-3} = \frac{4}{-1} = -4$

$m_{CD} = \frac{3-4}{-2-2} = \frac{1}{4}$

$m_{AD} = \frac{3-(-1)}{-2-(-1)} = \frac{4}{-1} = -4$



Since, $AB \parallel CD, BC \parallel AD$ and $AB \perp BC, AD$ $\quad CD \perp BC, AD$

$\therefore ABCD$ is a rectangle

Homework:

4.4 # 1-3 bcf…, 5, 6, 9, 10, 13

4.5 Applications of Linear Relations

Rate of change is the slope in word problems.
Fixed amount/cost is the y-intercept in word problems.

Ex. A plumbing service company charges a fixed amount, plus an hourly rate for a service call. A two-hour service is \$260, and a four-hour service call is \$440.

a. Write the equation that shows how the total cost, C, depends on the number of hours, t, and the fixed cost, b. Use r for hourly rate.

$$C = r \cdot t + b \qquad \rightarrow \qquad C = rt + b$$

Treat t and C, as x and y $\quad (x, y) \rightarrow (t, C)$

b. Find the hourly rate.

$$(2, 260), (4, 440)$$

$$m = \frac{440-260}{4-2} = \frac{180}{2} = 90$$

$\therefore r = 90$, the hourly rate is \$90/hr

c. Find the fixed amount / cost.

$$C = 90t + b$$

Substitute in one of the coordinates, either $(2, 260)$ or $(4, 440)$

$$260 = 90(2) + b$$

$$260 = 180 + b$$

$$b = 80$$

$\therefore$ the fixed cost is \$80

d. Determine the cost of an 8-hour job.

$$C = 90t + 80$$

$$C = 90(8) + 80$$

$$C = 720 + 80$$

$$C = 800$$

∴ the cost of an 8-hour job is $800

e. Determine the number of hours worked if the job costs $1880

$$90t + 80 = 1880$$

$$90t = 1800$$

$$t = 20$$

∴ a 20-hour job would cost $1880

Ex. A spring that is 24 inches long is compressed to 20 inches by a force of 16 pounds, and to 15 inches by a force of 36 pounds.

a. Determine the linear relation between the length of the spring, L, and the force, F.
Without any force applied, the spring is 24 in; this is the fixed amount.
$L = kF + 24$ (this is similar to $y = mx + b$)

Using $(16, 20)$ and $(36, 15)$, determine the slope.

$$k = \frac{15-20}{36-16} = \frac{-5}{20} = -\frac{1}{4}$$

∴ the spring will compress by 1 inch for every 4 pounds of force that is applied

$$L = -\frac{1}{4}F + 24$$

b. When a force of 28 pounds is applied, what is the length of the spring?

$$L = -\frac{1}{4}(28) + 24$$

$$L = -7 + 24$$

$$L = 17$$

∴ The length of the spring is 17 inches when 28 pounds of force is applied

c. Determine the amount of force required to compress the spring down to a length of 10 in.

$$10 = -\frac{1}{4}F + 24$$

$$-14 = -\frac{1}{4}F$$

$$F = 56$$

∴ The force applied is 56 pounds

Homework

4.5 # 1-11 odd

Next class

4.6 Chapter Review

#1-6 bcf, 7-14