Chapter 1 - Real Numbers

1.1 Number Systems

Number System Hierarchy

- Complex Numbers, $\mathbb{C}$
  - **Real,** $\mathbb{R}$
    - Rational, $\mathbb{Q}$
      - Integers, $\mathbb{Z}$
        - Whole, $\mathbb{W}$
          - Natural Numbers, $\mathbb{N}$
    - Irrational, $\overline{\mathbb{Q}}$
  - Imaginary, i

Complex Number System

The **complex number system** extends the **real number system** to include solutions to equations that have no real solutions—such as the square root of a negative number.

What is a Complex Number?

A complex number is a number in the form: $z = a + bi$

For the complex number $a + bi$, a is called the **real part**, and b is called the **imaginary part**. i is called the imaginary unit where $i^2 = -1$.

Although the main focus is on the real number system, a basic understanding of imaginary numbers (and complex numbers) will help explain some results that we will encounter later on.

Real Numbers, $\mathbb{R}$

The set of **real numbers**, $\mathbb{R}$, is composed of **rational numbers** and **irrational numbers.**

- Each real number corresponds to one point on a real number line

Rational Numbers, $\mathbb{Q}$

R**ational numbers**, $\mathbb{Q}$, is the set of numbers that:

- are terminating decimal numbers
- are non-terminating, repeating decimal numbers

Rational numbers are usually referred to as numbers that can be written as a fraction, in the form $\frac{a}{b}$ where a and b are integers, and $b \neq 0$

Ex. Write the following rational numbers as fractions in the form $\frac{a}{b}$

5, 7.2, $\sqrt{36}$, $\frac{3.5}{4}$, $2.\overline{13}$.

$5 = \frac{5}{1}$ $\qquad 7.2 = \frac{72}{10} = \frac{36}{5}$ $\qquad \sqrt{36} = 6 = \frac{6}{1}$

$\frac{3.5}{4} = \frac{7}{8}$ $\qquad 2.\overline{13} = 2\frac{13}{99} = \frac{211}{99}$

Irrational Numbers, $\overline{\mathbb{Q}}$

Irrational numbers, $\overline{\mathbb{Q}}$, is the set of numbers that are **non**-terminating, **non**-repeating decimal numbers.

- Irrational numbers **cannot** be written as a fraction in the form $\frac{a}{b}$ where a and b are $\mathbb{Z}$, and $b \neq 0$

So basically, irrational numbers are real numbers that are not rational.

Examples of irrational numbers

$\pi = 3.1415926\ldots$ $\sqrt{24} = 4.8989794\ldots$

$e = 2.7182818\ldots$ $\sqrt[3]{18} = 2.6207413\ldots$

Integers, $\mathbb{Z}$

Integers, $\mathbb{Z}$, is the set of positive and negative whole numbers, and also including zero.

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

Whole Numbers, $\mathbb{W}$

Whole numbers, $\mathbb{W}$, is the set of whole numbers

$$\mathbb{W} = \{0, 1, 2, 3, \ldots\}$$

Natural Numbers, $\mathbb{N}$

Natural Numbers, $\mathbb{N}$, is the set of whole numbers **not including 0**; also known as "counting numbers"

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

Ex. Name the set of numbers to which each number belongs.

a. $\sqrt{81}$

$= 9$

$\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

b. $\sqrt{3.6}$

$\approx 1.8973\ldots$

$\overline{\mathbb{Q}}, \mathbb{R}$

c. $-\sqrt{\frac{1}{4}}$

$= -\frac{1}{2}$

$\mathbb{Q}, \mathbb{R}$

d. $\sqrt{-0.36}$

$= \sqrt{0.36}\sqrt{-1} = 0.6i$

i

Real Number Operations

The most common operations that students have encountered up to now are addition, subtraction, multiplication, division, and exponents. There is another useful operation that will occur periodically, **absolute value**.

What is absolute value?

The **absolute value** of a real number is the distance from the number to zero. Since distances cannot be negative, the absolute value of a real number is always positive.

Ex. Simplify the following.

a. $|7|$

$= 7$

b. $|0.15|$

$= 0.15$

c. $|-15|$

$= 15$

d. $\left|-\frac{22}{9}\right|$

$= \frac{22}{9}$

e. $|5-9|$

$= |-4|$

$= 4$

f. $|7|-12$

$= 7-12$

$= -5$

1.1 Homework

1 bcf…, 2-4 bcf, 5-6 bcf…

1.2 GCF and LCM

Prime and Composite Numbers

Prime numbers are numbers that can be divisible by only 1 and itself

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, …

Composite numbers are whole numbers that has a divisor other than 1 and itself

4, 25, 100, 63, …

Zero and **one** are neither prime nor composite

Ex. Determine if it is a composite number or not

a. 3

it is prime, therefore not composite

b. 10

it is composite, divisible by 1, 2, 5 and 10

c. 18

it is composite, divisible by 1, 2, 3, 6, 9 and 18

d. 23

it is prime, therefore not composite

Prime Factorization

Prime factorization is the process of re-writing a number as a product of prime numbers only.

Ladder and tree methods can be used for prime factorization

Ex. Write the prime factorization of 24

24	12	6	3	1
2	2	2	3	

$24 = 2 \times 2 \times 2 \times 3$ or $24 = 2^3 \times 3$

Ex. Write the prime factorization of 60

60	30	15	5	1
2	2	3	5	

$60 = 2 \times 2 \times 3 \times 5$ or $60 = 2^2 \times 3 \times 5$

GCF – Greatest Common Factor

GCF is the largest number that goes into 2 or more numbers evenly.

Ex. Determine GCF of 24 and 60.

One approach is to re-write both 24 and 60 with prime factorization and then find the factors that exist in both numbers.

$24 = 2 \times 2 \times 2 \times 3$
$60 = 2 \times 2 \times 3 \times 5$

The GCF is the product of the shared common factors.

∴ GCF = $2 \times 2 \times 3 = 12$

This can be done by the ladder method on both numbers as well.

Ex. Determine GCF of 100 and 120.

100	50	25	5
120	60	30	6
2	2	5	

∴ GCF = $2 \times 2 \times 5 = 20$

Ex. Determine GCF of 32, 80, 128

32	16	8	4	2
80	40	20	10	5
128	64	32	16	8
2	2	2	2	

∴ GCF = $2 \times 2 \times 2 \times 2 = 16$

LCM – Least Common Multiple

The smallest number that divides evenly by a group of numbers.

Ex. Determine LCM of 15 and 18

One approach is to list multiples of each number until a common number appears for both numbers

15 = 15, 30, 45, 60, 75, 90
18 = 18, 36, 54, 72, 90

∴ The LCM = 90

Ex. Determine the LCM of 60 and 72

60	72	2
30	36	2
15	18	3
5	6	

The LCM is product of the all the numbers in the outside column and bottom row.

∴ LCM = 2 x 2 x 3 x 6 x 5 = 360

Ex. Determine the LCM of 42 and 56

42	56	2
21	28	7
3	4	

∴ LCM = 2 x 3 x 4 x 7 = 168

Ex. Determine the LCM of 27, 30, 36

Note: when determining the LCM of 3 or more numbers, the common factor needs only to apply to 2 of the numbers

27	30	36	2
27	15	18	3
9	5	6	3
3	5	2	

∴ LCM = 2 x 3 x 3 x 3 x 5 x 2 = 540

Ex. Determine the LCM of 8, 14, 15

8	14	15	2
4	7	15	

∴ LCM = 2 x 4 x 7 x 15 = 840

1.2 Homework

2 bcfg, 6 bcf…, 8 bcf…, 9 bcf…, 10 bcf…, 12, 15, 16, 19

1.3 Squares and Square Roots

Note: Roots and Radicals are used to refer to the same thing

Perfect Squares and Square Roots

In the statement, $5^2 = 25$,

5^2 is the **squared number** while 25 is the **perfect square.**

Square Roots of Perfect Squares

We know $\sqrt{25} = 5$, but why?

Recall, $\sqrt{25}$ is the same as $\sqrt{5^2}$ because $25 = 5^2$

A "square" and a square root are inverse operations; they cancel each other out

$\sqrt{25}$

$= \sqrt{5^2}$

$= 5$

Alternatively,

$\sqrt{25}$

$= \left(\sqrt{5}\right)^2$

$= 5$

Ex. Simplify $\sqrt{16x^4y^2}$

$= \sqrt{16} \cdot \sqrt{x^4} \cdot \sqrt{y^2}$

$= 4x^2y$

Perfect cubes and cube roots

In the statement $4^3 = 64$,
4^3 is the **cubed number** while 64 is the **perfect cube**.

Why is $\sqrt[3]{64}$ is equal to 4?

Similar to squares and square roots, a "cube" and cube root are inverse operations.

$\sqrt[3]{64}$

$= \sqrt[3]{4^3}$

$= 4$

or

$\sqrt[3]{64}$

$= \left(\sqrt[3]{4}\right)^3$

$= 4$

Ex. Simplify $\sqrt[3]{125x^9y^{12}}$

$= \sqrt[3]{125} \cdot \sqrt[3]{x^9} \cdot \sqrt[3]{y^{12}}$

$= 5x^3y^4$

Using prime factorization to evaluate square roots of perfect squares

Ex. Evaluate $\sqrt{144}$ using prime factorization.

144	72	36	18	9	3	1
2	2	2	2	3	3	

$\sqrt{144} = \sqrt{2^2} \times \sqrt{2^2} \times \sqrt{3^2}$

$= 2 \times 2 \times 3 = 12$

Ex. Evaluate $\sqrt[3]{64}$ using prime factorization

64	32	16	8	4	2	1
2	2	2	2	2	2	

$\sqrt[3]{64} = \sqrt[3]{2^3} \times \sqrt[3]{2^3}$

$= 2 \times 2 = 4$

Evaluate by factorizing a radical

Ex. Evaluate $-\sqrt{72900}$ without a calculator

$= -\sqrt{729}\sqrt{100}$

$= -\sqrt{81}\sqrt{9}\sqrt{100}$

$= -(9)(3)(10)$

$= -270$

Ex. Evaluate $\sqrt[3]{216000}$ without a calculator

$= \sqrt[3]{216}\sqrt[3]{1000}$

$= (6)(10)$

$= 60$

Ex. Evaluate $\sqrt{0.0729}$ without a calculator

$= \sqrt{729} \cdot \sqrt{0.0001}$

$= 27 \cdot 0.01$

$= 0.27$

Ex. Evaluate $\sqrt[3]{0.216}$ without a calculator

$$= \sqrt[3]{216} \cdot \sqrt[3]{0.001}$$

$$= 6 \cdot 0.1$$

$$= 0.6$$

Ex. A sphere has a volume of 288π ft^3. Determine the diameter of the sphere.

$$V = \frac{4}{3}\pi r^3 \qquad V = 288\pi$$

$$\frac{4}{3}\pi r^3 = 288\pi$$

$$\frac{4}{3}r^3 = 288$$

$$\frac{3}{4} \times \frac{4}{3}r^3 = 288 \times \frac{3}{4}$$

$$r^3 = 216$$

$$r^3 = 6^3$$

$$\sqrt[3]{r^3} = \sqrt[3]{6^3}$$

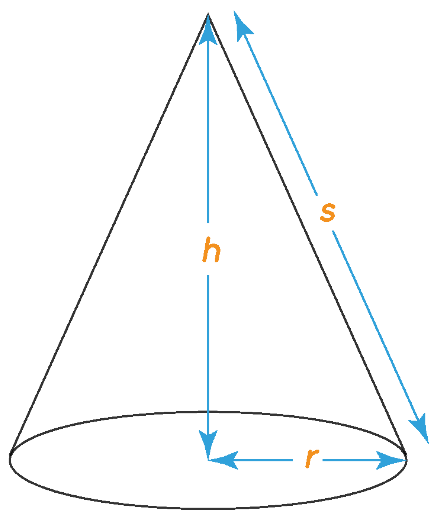
$$r = 6$$

$$\therefore d = 2r = 12$$

∴ the diameter of the sphere is 12 ft

Ex. A right-angle cone has a volume of 100π cm^3. The ratio of the diameter to the height cone is 5:6. Determine the surface area of the cone.

$V = \frac{1}{3}\pi r^2 h$ $\quad$ $V = 100\pi$ $\quad$ $SA = \pi rs + \pi r^2$



$d: h = 5: 6$

$2r: h = 5: 6$

$\frac{2r}{h} = \frac{5}{6}$

$5h = 12r$

$h = \frac{12}{5}r$

$\frac{1}{3}\pi r^2 h = 100\pi$

$\frac{1}{3}\pi r^2 \left(\frac{12}{5}r\right) = 100\pi$

$\frac{4}{5}r^3 = 100$

$r^3 = 125$

$\sqrt[3]{r^3} = \sqrt[3]{5^3}$

$r = 5$

$\therefore h = \frac{12}{5}(5) = 12$

Use Pythagorean Theorem to calculate the slant height s.

$s^2 = 5^2 + 12^2$

$s^2 = 169$

$s = \pm 13$, reject the negative

$s = 13$

$SA = \pi r s + \pi r^2$

$= \pi(5)(13) + \pi(5)^2$

$= 65\pi + 25\pi$

$= 90\pi$ cm^2

$\therefore$ the surface area of the cone is 90π cm^2

1.3 Homework

1-4 bcf…, 5, 7, 9, 10

1.4 Rational and Irrational Numbers

Roots of Non-Perfect Squares and Non-Perfect Cubes

We will investigate how to deal with roots of non-perfect squares and non-perfect cubes.

We will look at two methods of dealing with irrational numbers: **approximation** and **simplifying** (exact value). Section 1.4 will investigate approximation, while Section 1.6 looks at simplifying.

Linear Approximation (from grade 8 and 9)

Ex. Estimate $\sqrt{45}$ to 1 decimal place using linear approximation.

Since $\sqrt{45}$ is between $\sqrt{36}$ and $\sqrt{49}$, we can conclude that the approximate value of $\sqrt{45}$ is between 6 and 7.

$$\therefore \sqrt{45} \approx 6.\_\_$$

To approximate the decimal place, we need to do the following calculation:

$$\frac{45-36}{49-36} = \frac{9}{13}$$

$$\frac{9}{13} = 0.6923 \ldots \approx 0.7$$

$$\therefore \sqrt{45} \approx 6.7$$

We apply similar principles from the example above to cube roots.

Ex. Approximate $\sqrt[3]{100}$ to 1 decimal place using linear approximation.

$\sqrt[3]{100}$ is between $\sqrt[3]{64}$ and $\sqrt[3]{125}$

$\therefore \sqrt[3]{100} \approx 4.\_\_$

$\frac{100-64}{125-64} = \frac{36}{61}$

$\frac{36}{61} = 0.5901 \ldots \approx 0.6$

$\therefore \sqrt[3]{100} \approx 4.6$

Using Approximate Values of Radicals to Approximate Similar Radicals

Given $\sqrt{54} \approx 7.35$ and $\sqrt{540} \approx 23.24$, determine the value of the following square roots.

Ex. Estimate $\sqrt{5400}$

$= \sqrt{54} \times \sqrt{100}$

$= 7.35 \times 10 = 73.5$

Ex. Estimate $\sqrt{5.4}$

$= \sqrt{540} \times \sqrt{0.01}$

$= 23.24 \times 0.1 = 2.324$

Given $\sqrt[3]{24} \approx 2.88$, $\sqrt[3]{240} \approx 6.21$ and $\sqrt[3]{2400} \approx 13.39$, determine the value of the following cube roots.

Ex. Estimate $\sqrt[3]{0.024}$

$= \sqrt[3]{24} \times \sqrt[3]{0.001}$

$= 2.88 \times 0.1 = 0.288$

Ex. Estimate $\sqrt[3]{240000}$

$= \sqrt[3]{240} \times \sqrt[3]{1000}$

$= 6.21 \times 10 = 62.1$

Ex. Estimate $\sqrt[3]{2.4}$

$= \sqrt[3]{2400} \times \sqrt[3]{0.001}$

$= 13.39 \times 0.1 = 1.339$

1.4 Homework

1-5 bcf…, 6 ace…, 7 bcf…, 8 ac

1.5 Exponential Notation

Recall: 5^3 is the same as $5 \times 5 \times 5$

$(4x)^2$ is the same as $4x \cdot 4x$

$(-2y)^3$ is the same as $(-2y)(-2y)(-2y)$

$12x^2y^3$ is the same as $12(x)(x)(y)(y)(y)$

One and Zero Exponent

Any number to the power of 0 is equal to 1, and any number to the power of 1 equals itself.

$(12)^1 = 12$ $(12)^0 = 1$ $-(12)^0 = -1$

$(3\pi)^1 = 3\pi$ $(-3\pi)^0 = 1$ $-(3\pi)^0 = -1$

Exponent Laws

The Product Rule: $b^m \times b^n = b^{m+n}$

Ex. Simplify $x^4 \cdot x^5$

$= x^{4+5}$

$= x^9$

Ex. Simplify $2^{2x} \times 2^{3x}$

$= 2^{2x+3x}$

$= 2^{5x}$

Ex. Simplify $8(2^{2x})(2^3)$

$= 2^3(2^{2x})(2^3)$

$= 2^{3+2x+3}$

$= 2^{2x+6}$

The Quotient Rule: $\frac{b^m}{b^n} = b^{m-n}$ $b^m \div b^n = b^{m-n}$

Ex. Simplify $\frac{x^7}{x^4}$

$= x^{7-4}$

$= x^3$

Ex. Simplify $\frac{x^6y^8}{x^4y}$

$= x^{6-4}y^{8-1}$

$= x^2y^7$

Ex. Simplify $\frac{x^5yz^3}{x^2y^3z^3}$

$= x^3y^{-2}z^0$

$= \frac{x^3}{y^2}$

The Power Rule: $(b^m)^n = b^{mn}$

Ex. Simplify $(x^5)^4$

$= x^{4\cdot 5}$

$= x^{20}$

Ex. Simplify $[(g^2)^3]^6$

$= (g^2)^{18}$

$= g^{36}$

Ex. Simplify $2^{2x}(16^{3x})$

$= 2^{2x}[(2^4)^{3x}]$

$= 2^{2x}[2^{12x}]$

$= 2^{14x}$

Ex. Simplify $81^{6x} \div 27^{5x}$

$= (3^4)^{6x} \div (3^3)^{5x}$

$= 3^{24x} \div 3^{15x}$

$= 3^{9x}$

Raising a Product to a Power: $(a^x b^y)^n = a^{nx} b^{ny}$

Ex. Simplify $(5^2 x^3)^2$

$= 5^{2\times 2} x^{3\times 2}$

$= 5^4 x^6$ or $= 625x^6$

Ex. Simplify $(2x^3 y^2)^4$

$= 2^{1\times 4} x^{3\times 4} y^{2\times 4}$

$= 16x^{12} y^8$

Ex. Simplify $(-6a^3 b)^2$

$= (-6)^2 a^{3\times 2} b^{1\times 2}$

$= 36a^6 b^2$

Ex. Simplify $\left(\frac{2x}{3y}\right)^2$

$= \frac{2^2 x^2}{3^2 y^2}$

$= \frac{4x^2}{9y^2}$

Negative Exponents $b^{-m} = \frac{1}{b^m}$ $\left(\frac{b^m}{a^n}\right)^{-1} = \frac{a^n}{b^m}$ $\left(\frac{b^m}{a^n}\right)^{-x} = \left(\frac{a^n}{b^m}\right)^x$

Ex. Simplify 5^{-3}

$= \frac{1}{5^3}$

$= \frac{1}{125}$

Ex. Simplify $\left(\frac{2x^3}{3y^2}\right)^{-2}$

$= \left(\frac{3y^2}{2x^3}\right)^2$	$= \frac{2^{-2}x^{-6}}{3^{-2}y^{-4}}$
$= \frac{3^2y^{2\times2}}{2^2x^{3\times2}}$	$= \frac{3^2y^4}{2^2x^6}$
$= \frac{9y^4}{4x^6}$	$= \frac{9y^4}{4x^6}$

Note: do not leave final answer with negative exponents

$3x^2y^{-3} \rightarrow \frac{3x^2}{y^3}$ $\frac{25a}{16b^{-3}} \rightarrow \frac{25ab^3}{16}$

Rational Exponents: $b^{\frac{1}{n}}$

$b^{\frac{1}{n}} = \sqrt[n]{b}$ n is a positive integer

Ex. Write $x^{\frac{1}{2}}$ as a radical

$= \sqrt{x}$ or $= \sqrt[2]{x}$

Ex. Write $\sqrt[4]{6}$ as an exponential

$= 6^{\frac{1}{4}}$

Multiplying Radicals with Different Index Numbers

Ex. Simplify $\sqrt{2} \times \sqrt[4]{2}$

Cannot multiply two radicals with different index numbers; convert to exponential form and then simplify. Leave answer as a radical.

$= 2^{\frac{1}{2}} \times 2^{\frac{1}{4}}$ $\sqrt{2} = \sqrt[2]{2^1}$

$= 2^{\frac{1}{2}+\frac{1}{4}}$

$= 2^{\frac{3}{4}}$

$= \sqrt[4]{2^3}$ or $\sqrt[4]{8}$

Ex. Simplify $\sqrt[3]{3} \times \sqrt{3}$

$= 3^{\frac{1}{3}} \times 3^{\frac{1}{2}}$

$= 3^{\frac{1}{3}+\frac{1}{2}}$

$= 3^{\frac{5}{6}}$

$= \sqrt[6]{3^5}$ or $= \sqrt[6]{243}$

Rational Exponents: $a^{\frac{m}{n}}$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \qquad \text{or} \qquad \left(\sqrt[n]{a}\right)^m$$

Ex. Simplify $8^{\frac{4}{3}}$

$= \sqrt[3]{8^4}$ or $= \left(\sqrt[3]{8}\right)^4$

$= \sqrt[3]{4096} = 16$ $\qquad$ $= (2)^4 = 16$

Ex. Simplify $4^{\frac{3}{2}}$

$= \left(\sqrt{4}\right)^3$ or $= (2^2)^{\frac{3}{2}}$

$= (2)^3 = 8$ $\qquad$ $= 2^3 = 8$

Ex. Simplify $\sqrt[3]{9} \times \sqrt{27}$

$= \sqrt[3]{3^2} \times \sqrt{3^3}$

$= 3^{\frac{2}{3}} \times 3^{\frac{3}{2}}$

$= 3^{\frac{2}{3}+\frac{3}{2}}$

$= 3^{\frac{13}{6}}$

$= \sqrt[6]{3^{13}}$ or $= 9\sqrt[6]{3}$

Ex. Simplify $\frac{\sqrt[3]{4}}{\sqrt[4]{2}}$

$$= \frac{\sqrt[3]{2^2}}{\sqrt[4]{2^1}}$$

$$= \frac{2^{\frac{2}{3}}}{2^{\frac{1}{4}}}$$

$$= 2^{5/12}$$

$$= \sqrt[12]{2^5}$$

$$= \sqrt[12]{32}$$

Ex. Simplify $\sqrt[4]{25}$

$$= \sqrt[4]{5^2}$$

$$= 5^{\frac{2}{4}}$$

$$= 5^{\frac{1}{2}}$$

$$= \sqrt{5}$$

1.5 Homework

#1-11 bcf…

1.6 Irrational Numbers

Rational Numbers

A number that can be written in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

-> integers, terminating decimals, non-terminating repeating decimals

Examples of rational numbers: $5 = \frac{5}{1}$ $0.3 = \frac{3}{10}$ $0.333 \ldots = \frac{1}{3}$

Irrational Numbers

Real numbers that **cannot** be written as a fraction $\frac{a}{b}$ where $a, b \in \mathbb{Z}$

-> non-terminating, non-repeating decimals

Examples of irrational numbers: $\sqrt{12}$ π $10\sqrt{3}$

Instead of approximating radicals, we can simplify them and keep the **exact value** of the radical.

Radical: $\sqrt[n]{x}$

n - the **Index Number**, indicates the root of the radical

x - the **Radicand**, the number or expression inside the radical

Radicals can be written as a **Mixed Radical** or an **Entire Radical**

Mixed radical examples: $2\sqrt{6}$, $-5\sqrt{2}$, $3\sqrt{56}$

Entire radical example: $\sqrt{24}$, $-\sqrt{50}$, $\sqrt{504}$

Converting Mixed Radical to an Entire Radical

Ex. Write $-5\sqrt{6}$ as an entire radical.

$= -\sqrt{5^2} \cdot \sqrt{6}$

$= -\sqrt{25} \cdot \sqrt{6}$

$= -\sqrt{150}$

Ex. Write $3\sqrt[3]{4}$ as an entire radical.

$= \sqrt[3]{3^3} \cdot \sqrt[3]{4}$

$= \sqrt[3]{27} \cdot \sqrt[3]{4}$

$= \sqrt[3]{108}$

Ex. Write $3x^2y\sqrt{5xy}$ as an entire radical.

$= \sqrt{(3x^2y)^2} \cdot \sqrt{5xy}$

$= \sqrt{9x^4y^2} \cdot \sqrt{5xy}$

$= \sqrt{45x^5y^3}$

Simplifying Radicals

When a radical is in simplest form:

- the radicand has no perfect square factors other than 1

- the radicand does not contain a fraction

- no radical appears in the denominator

Ex. Simplify $\sqrt{56}$

Method 1: using prime factorization

56 = 2 x 2 x 2 x 7

Because the index is **2**, we group by **pairs**

All **pairs** of numbers come out of the radical, while the non-paired stays inside the radicand

$$\sqrt{56} = 2\sqrt{2 \times 7}$$

$$= 2\sqrt{14}$$

Method 2: factor the largest perfect square from the radicand

$$\sqrt{56} = \sqrt{4} \times \sqrt{14}$$

$$= 2\sqrt{14}$$

Ex. Simplify $5\sqrt{216}$

Method 1:

216	108	54	27	9	3	1
2	2	2	3	3	3	

$5\sqrt{216}$

$= 5 \times 2 \times 3\sqrt{2 \times 3}$

$= 30\sqrt{6}$

Method 2:

$5\sqrt{216}$

$= 5 \cdot \sqrt{36} \cdot \sqrt{6}$

$= 5 \cdot 6 \cdot \sqrt{6}$

$= 30\sqrt{6}$

Radical Properties

Multiplication $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ where $a \geq 0$ and $b \geq 0$

Ex. Simplify $\sqrt{3} \times \sqrt{2}$

$= \sqrt{3 \times 2}$

$= \sqrt{6}$

Ex. Simplify $\sqrt{5} \times \sqrt{7} \times \sqrt{3}$

$= \sqrt{5 \times 7 \times 3}$

$= \sqrt{105}$

Ex. Simplify $\sqrt{15} \times \sqrt{12}$

$= \sqrt{180}$

$= \sqrt{36} \cdot \sqrt{5}$

$= 6\sqrt{5}$

Ex. Simplify $\sqrt[3]{4} \times \sqrt[3]{15}$

$= \sqrt[3]{4 \times 15}$

$= \sqrt[3]{60}$

Ex. Simplify $\sqrt[3]{14} \times \sqrt[3]{20}$

$= \sqrt[3]{280}$

$= \sqrt[3]{8} \cdot \sqrt[3]{35}$

$= 2\sqrt[3]{35}$

Extend the Multiplication Rule: $\boldsymbol{a\sqrt{x}\left(b\sqrt{y}\right) = ab\sqrt{xy}}$

The coefficients are multiplied together, while the radicands are multiplied together

Ex. Simplify $2\sqrt{15} \times 3\sqrt{6}$

$= 6\sqrt{90}$

$= 6\sqrt{9} \times \sqrt{10}$

$= 6 \times 3\sqrt{10}$

$= 18\sqrt{10}$

Ex. Simplify $\sqrt{3} \times \sqrt[3]{2}$

$= 3^{\frac{1}{2}} \times 2^{\frac{1}{3}}$

$= 3^{\frac{3}{6}} \times 2^{\frac{2}{6}}$

$= \sqrt[6]{3^3} \times \sqrt[6]{2^2}$

$= \sqrt[6]{27}\sqrt[6]{4}$

$= \sqrt[6]{108}$

Division $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where $a \geq 0$ and $b > 0$

Ex. Simplify $\sqrt{75} \div \sqrt{25}$

$= \sqrt{\frac{75}{25}}$

$= \sqrt{3}$

Ex. Simplify $\frac{\sqrt[3]{96}}{\sqrt[3]{6}}$

$= \sqrt[3]{\frac{96}{6}}$

$= \sqrt[3]{16}$

$= \sqrt[3]{8} \cdot \sqrt[3]{2}$

$= 2\sqrt[3]{2}$

1.6 Homework

1-10 (bcf…), 12, 13, 16, 17, 20

Radicals with Fractions in the Radicand or Radical in the Denominator

Ex. Simplify $\frac{\sqrt{18}}{2\sqrt{6}}$

$= \frac{\sqrt{3}}{2\sqrt{1}}$

$= \frac{\sqrt{3}}{2}$

If a radical appears in the denominator, you need to go through the process of **Rationalizing the Denominator**

Ex. Simplify $\frac{9}{\sqrt{12}}$

$= \frac{9}{2\sqrt{3}}$

$= \frac{9}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{9\sqrt{3}}{2 \cdot 3}$

$= \frac{9\sqrt{3}}{6}$

$= \frac{3\sqrt{3}}{2}$

Ex. Simplify $\sqrt{\frac{3}{5}}$

$$= \frac{\sqrt{3}}{\sqrt{5}}$$

$$= \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{15}}{5}$$

Ex. Simplify $\frac{5}{\sqrt{3}}$

$$= \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{3}$$

Ex. Simplify $\frac{4\sqrt{2}}{3\sqrt{5}}$

$$= \frac{4\sqrt{2}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{4\sqrt{10}}{3 \cdot 5}$$

$$= \frac{4\sqrt{10}}{15}$$

Ex. Simplify $\frac{12\sqrt{75}}{8\sqrt{6}}$

$$= \frac{3\sqrt{25}}{2\sqrt{2}}$$

$$= \frac{3 \cdot 5}{2\sqrt{2}}$$

$$= \frac{15}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{15\sqrt{2}}{4}$$

Ex. Simplify $\frac{1}{\sqrt[3]{2}}$

Note: $\sqrt[3]{2} \cdot \sqrt[3]{2} = \sqrt[3]{2^2} \neq 2$

instead $\sqrt[3]{2} \cdot \sqrt[3]{2^2} = \sqrt[3]{2^3} = 2$

$$= \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$$

$$= \frac{\sqrt[3]{4}}{2}$$

Homework:
Simplifying Radicals Worksheet #1-55

Review Assignment