Trigonometric Functions Test

Name: Answer Key / 40

[2] 1. Determine two co-terminal angles with the given θ. Round to 3 decimal places if necessary.

[1] a. $324°$

$= 324° - 360°$ $= 324° + 360°$

$= -36°$ $= 684°$

[1] b. 0.123

$= 0.123 - 2\pi$ $= 0.123 + 2\pi$

$= -6.160$ $= 6.406$

[2] 2. Determine the quadrant in which θ lies for the following.

[1] a. $\sec\theta < 0, \tan\theta > 0$

$\cos\theta < 0$ in QII and QIII

$\tan\theta > 0$ in QI and QIII

$\therefore$ QIII

[1] b. $\sin\theta > 0, \csc\theta < 0$

$\sin\theta > 0$ in QI and QII

$\csc\theta < 0$ in QIII and QIV

$\therefore$ none

[2] 3. Determine the coordinates of the point at the given distance from the origin in the stated quadrant, if θ is its position angle.

[1] a. 15, quadrant II, $\cos\theta = -\frac{4}{5}$

$\cos\theta = \frac{x}{15}$ $\rightarrow$ $\frac{x}{15} = -\frac{4}{5}$

$x = -12$

$y = \sqrt{15^2 - (-12)^2} = 9$

$\therefore (-12, 9)$

[1] b. $6x$, quadrant III, $\tan\theta = 1$

Let (a, b) be the coordinate

$\tan\theta = \frac{b}{a}$ $\tan\theta = 1$ $\therefore a = b$

$a^2 + b^2 = (6x)^2$ since $a = b$

$2a^2 = 36x^2$

$a^2 = 18x^2$ $\rightarrow$ $a = \pm 3\sqrt{2}x$

(a, b) in QIII so $\left(-3\sqrt{2}x, -3\sqrt{2}x\right)$

[2] 4. For an arc with a radius of 1 cm and sector area of $\frac{\pi}{5}$ cm^2, determine the arc length of the sector.

$A = \frac{r^2\theta}{2}$ $s = r\theta$

$\frac{\pi}{5} = \frac{(1)^2\theta}{2}$ $s = (1)\left(\frac{2\pi}{5}\right)$

$\theta = \frac{2\pi}{5}$ $s = \frac{2\pi}{5}$ cm

[3] 5. Find all θ, $0 \le \theta < 2\pi$ for $\cos\theta = -\frac{1}{\sqrt{3}}$. Round to 3 decimal places if necessary.

θ in QII and QIII

$\theta_1 = \cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) = 2.1862\ldots$ $\theta_r = \pi - 2.1862\ldots = 0.9553\ldots$

$\theta_2 = \pi + 0.9553\ldots = 4.0968\ldots$

$\therefore \theta = 2.186, 4.097$

[3] 6. Find all θ, $0° \le \theta \le 360°$ for $-\tan\theta + 2 = \tan\theta$. Round to 1 decimal place if necessary.

$2\tan\theta = 2$

$\tan\theta = 1$ θ is in QI and QIII

$\theta_1 = 45°$ $\theta_2 = 180° + 45° = 225°$

$\therefore \theta = 45°, 225°$

[4] 7. Determine the exact value of the following trigonometric functions.

[2] a. $\sin\frac{5\pi}{4}$

$\frac{5\pi}{4}$ in QIII where $\sin\theta < 0$

$\theta_r = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\therefore \sin\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$

[2] b. $\tan\frac{17\pi}{3}$

$\frac{17\pi}{3}$ is co-terminal with $\frac{5\pi}{3}$ which is in QIV where $\tan\theta < 0$

$\theta_r = 2\pi - \frac{5\pi}{3} = \frac{\pi}{3}$

$\tan\frac{\pi}{3} = \sqrt{3}$ $\therefore \tan\frac{17\pi}{3} = -\sqrt{3}$

[4] 8. Find the values of $\sin\theta$, $\cos\theta$, and $\cot\theta$ if θ is an angle in standard position whose terminal side is the graph $7x - y = 0, x \le 0$. Exact answers only.

$y = 7x$ and $x \le 0$ $\rightarrow$ $(-1, -7)$

$r = \sqrt{(-1)^2 + (-7)^2} = \sqrt{50} = 5\sqrt{2}$

$\sin\theta = -\frac{7}{5\sqrt{2}}$ $\cos\theta = -\frac{1}{5\sqrt{2}}$ $\cot\theta = \frac{-1}{-7} = \frac{1}{7}$

[8] 9. For a cosine function where a maximum value of 4 occurs at $x = \frac{\pi}{4}$ and the next minimum value of -2 occurs at $x = \frac{3\pi}{4}$

[1] a. Determine the period.

$$\frac{1}{2} period = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$$

$$\therefore period = \pi$$

[1] b. Determine the amplitude.

$$amp = \frac{4-(-2)}{2}$$

$$amp = 3$$

[1] c. Determine the phase shift.

$y = \cos x$ starts at a maximum

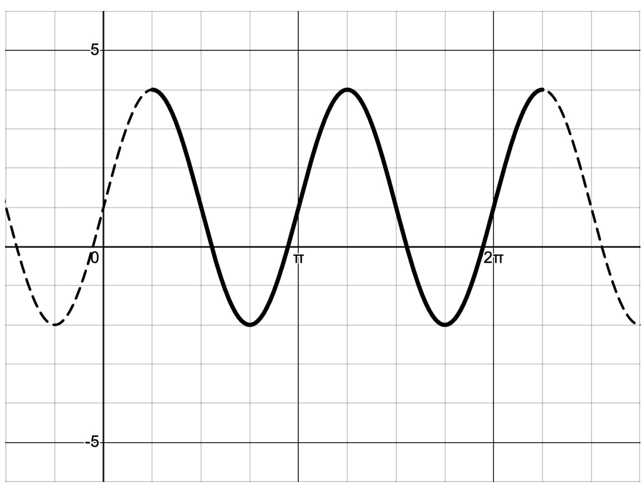
$$\therefore c = \frac{\pi}{4}$$

[1] d. Determine the vertical displacement.

$$d = \frac{4+(-2)}{2}$$

$$d = 1$$

[2] e. Graph 2 periods of the function that is described above. Include all appropriate labels.



[2] f. Write an equation in the form $y = a \sin b(x - c) + d$ for the least non-negative real number c, with $a > 0$ and $b > 0$ for the graph above.

$$c = \frac{\pi}{4} - \frac{1}{4} period$$

$$c = \frac{\pi}{4} - \frac{1}{4}\pi = 0$$

$$y = 3 \sin 2x + 1$$

[10] 10. Mr. Kwan's alertness level throughout the day can be modeled using a sinusoidal function. He has a maximum alertness level of 90% at 8:00 am and a minimal alertness level of 20% at 11:30 am.

[4] a. Write a sinusoidal function that describes Mr. Kwan's alertness level starting at 8:00 am.

$amp = \frac{0.9-0.2}{2} = 0.35$ $\quad$ $\frac{1}{2}period = 11.5 - 8 = 3.5 \quad \rightarrow period = 7 \;\therefore b = \frac{2\pi}{7}$

$c = 8$ is maximum, so use cosine $\quad$ $d = \frac{0.9+0.2}{2} = 0.55$

$y = 0.35\cos\frac{2\pi}{7}(t-8) + 0.55$

[1] b. Determine Mr. Kwan's alertness level at 7:00 am (assuming he is awake!). Round to the nearest percent.

7:00 am $\rightarrow t = 7$

$y = 0.35\cos\frac{2\pi}{7}(7-8) + 0.55 = 0.768 = 76.8\%$

[3] c. Determine the amount of time Mr. Kwan's alertness level is above 70% during the school day from 8:00 am to 4:00 pm. Round to the nearest minute.

$0.7 = 0.35\cos\frac{2\pi}{7}(t-8) + 0.55$

$0.15 = 0.35\cos\frac{2\pi}{7}(t-8)$

$\frac{3}{7} = \cos\frac{2\pi}{7}(t-8)$ $\quad$ solution in QI & QIV

$\frac{2\pi}{7}(t-8) = \cos^{-1}\left(\frac{3}{7}\right)$

$\frac{2\pi}{7}(t-8) = 1.128, 5.155$

$\frac{2\pi}{7}(t_1-8) = 1.128$ $\quad$ $\frac{2\pi}{7}(t_2-8) = 5.155$

$t_1 - 8 = 1.257$ $\quad$ $t_2 - 8 = 5.743$

$t_1 = 9.257$ $\quad$ $t_2 = 13.743$

$t = (9.257 - 8) + (16 - 13.743) = 3.514$

$t = 3$ hrs 31 minutes

[2] d. During the school day, at what time(s) will Mr. Kwan's alertness level be at 50%. Round to the nearest minute.

$0.5 = 0.35\cos\frac{2\pi}{7}(t-8) + 0.55$

$-0.05 = 0.35\cos\frac{2\pi}{7}(t-8)$

$-\frac{1}{7} = \cos\frac{2\pi}{7}(t-8)$ $\quad$ solution in QII & QIII

$\frac{2\pi}{7}(t-8) = \cos^{-1}\left(-\frac{1}{7}\right)$

$\frac{2\pi}{7}(t-8) = 1.714, 4.569$

$\frac{2\pi}{7}(t_1-8) = 1.714$ $\quad$ $\frac{2\pi}{7}(t_2-8) = 4.569$

$t_1 - 8 = 1.910$ $\quad$ $t_2 - 8 = 5.090$

$t_1 = 9.910$ $\quad$ $t_2 = 13.090$

$t_1 = 9{:}55$ am $\quad$ $t_2 = 1{:}04$ pm